Monetary Policy, Financial Constraints, and Redistribution[1]

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Abstract

This paper examines how financial constraints affect redistribution via monetary policy. We explore a novel mechanism of monetary non-neutrality, which is based on debt limits imposed in nominal terms. Specifically, when debt is constrained by current income, monetary policy can alter the real terms of borrowing. Changes in inflation exert ambiguous effects, depending on the initial debt/wealth position and the willingness to borrow. We show analytically that borrowers can benefit from increased debt limits under lower inflation rates. This novel effect can dominate conventional debt deflation effects. We find that particularly less indebted borrowers as well as potential future borrowers gain and that aggregate welfare can be enhanced under a permanent reduction in inflation.

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1 Introduction

Based on broad empirical evidence, the vast majority of studies on monetary policy effects considers nominal rigidities in goods and labor markets as the main sources of monetary non-neutrality. In contrast, the role of financial frictions in this regard has received much less attention in the literature, even though their existence is undisputed. Debt is typically issued in nominal terms and in a non state-contingent way, such that changes in the price level can alter real payoffs. This transmission channel of unexpected monetary policy (the so-called Fisher debt deflation channel) is well-established and has been examined in several studies.[3] In this paper, we examine a novel channel of monetary transmission via financial constraints expressed in nominal terms.[4] Thereby, monetary policy exerts redistributive effects, which complements other recently-studied general equilibrium effects of monetary policy on heterogeneous agents (see Kaplan and Violante, 2018, or Auclert, 2019).[5]

The central elements of our analysis are that only nominal debt is available and that outstanding debt is limited by current income. The latter assumption is motivated by empirical evidence provided by numerous studies that current income or earnings serve as a relevant limit for unsecured debt (see e.g., Japelli and Pagano, 1989, Jappelli, 1990, Duca and Rosenthal, 1993, Del Río and Young, 2006, Choi et al., 2018, Dettling and Hsu, 2018, Drechsel, 2019, or Lian and Ma, 2019) and by various theoretical studies that focus on current – rather than on future – factors that limit debt. In particular, studies on fire sales consider the impact of asset sales on their current period value (see e.g., Stein, 2012, Woodford, 2016, Davila and Korinek, 2018), inducing debt deleveraging by tightening the limit for end-of-period debt within the same period. Moreover, studies that rationalize macroprudential regulation by pecuniary externalities consider borrowing limits that restrict end-of-period debt by current period income valued at current

[3] For example, Doepke et al. (2015) or Auclert (2019) are recent contributions to this literature. They further provide comprehensive overviews over studies on distributional effects of monetary policy.

[4] Gariga et al. (2017) examine the transmission of monetary policy via nominal rigidities induced by fixed-rate and adjustable-rate mortgage contracts.

[5] Kaplan and Violante (2018) show that indirect (general equilibrium) effects via labor income of heterogeneous households can outweight direct effects of monetary policy, in particular, via intertemporal substitution.

relative prices (see e.g. Bianchi, 2011, Benigno et al., 2016, Korinek, 2018, or Schmitt-Grohe and Uribe, 2019).[6] This type of constraints can principally be rationalized by the inability of borrowers to commit to repay. If debt can be renegotiated after issuance, borrowers – who cannot commit – might make a take-it-of-leave-it offer to reduce the value of debt. Suppose that lenders who reject this offer, can seize borrowers' wealth. When assets are not available, as considered in this paper, an offer will thus be accepted when the repayment value of debt does not exceed borrowers' available income. Under a repudiation-proof debt contract, outstanding debt is then restricted by current income.[7]

We explore implications for monetary policy and its redistributive effects under fully flexible goods prices when repayment of unsecured debt is constrained by current income. Apparently, the debt limit in terms of commodities at maturity can then be affected by price level changes and thereby by monetary policy. To make this argument more transparent, consider a nominal repayment S_{t+1} that is contracted in t at the period t price Q_t and due in $t+1$. Suppose that it is limited at issuance by current income, $S_{t+1} \leq P_t y_t$, where y_t denotes an exogenous real income and P_t the price level in period t. Then, real debt repayment in terms of commodities in period $t+1$, $x_{t+1} = S_{t+1}/P_{t+1}$, has to satisfy $x_{t+1} \leq y_t/\pi_{t+1}$, where π_{t+1} denotes the inflation rate $\pi_{t+1} = P_{t+1}/P_t$. Thus, a change in the inflation rate alters the effective debt limit, i.e. the maximum debt in terms of commodities at maturity.[8]

To understand the macroeconomic effects of monetary policy under a debt limit based on current income, consider for example an unexpected permanent increase in the inflation rate. On the one hand, a higher inflation rate implies the debt limit to shrink in terms of commodities at maturity. Given that this reduction in the value of debt at maturity is internalized by lenders, they also demand a lower debt price Q_t at issuance, which tends to reduce the maximum amount of funds that can be borrowed. On the other hand,

[6] Other theoretical studies, which consider current income as debt limits are, for example, Laibson et al. (2003) or Mendoza (2006).

[7] Such a constraint relates total outstanding debt to income and therefore differs from payment-to-income ratios that are relevant for mortgage (see Corbae and Quintin, 2015, or Greenwald, 2018).

[8] If debt limits instead account for expected future price changes, debt limits would be specified in terms of commodities at maturity, implying that monetary policy does not affect the effective tightness of the borrowing constraints. Then, monetary policy matters just due to the (conventional) effects of initial debt deflation.

there is a beneficial effect of the reduced debt repayment value in terms of commodities at maturity, which is in fact identical to the conventional debt deflation effect in the initial period. Hence, the increase in inflation tends to reduce the maximum amount of debt that can be issued (*debt limit effect*) as well as the stock of debt to be repaid (*debt deflation effect*). The beneficial debt deflation effect is opposed to the impact on the effective debt limit. Thus, the effect of higher inflation on borrowers' overall consumption possibilities and welfare is ex-ante ambiguous, and particularly depends on the likelihood that the borrowing constraint is binding and on the borrowers' initial debt level. Moreover, when borrowing decreases due to a tighter effective debt limit under a higher inflation rate, the real interest rate and thus the real cost of borrowing tend to fall.[9]

To assess the overall impact of changes in the inflation rate, we examine two distinct economies. We first consider the highly stylized case of a stationary equilibrium of an economy where agents permanently differ by their degree of patience (as for example studied by Kiyotaki and Moore, 1997). Relatively impatient agents tend to frontload consumption and are willing to borrow from more patient agents up to the maximum amount. A higher inflation rate then leads to the two effects described above: Debt repayment as well as the amount of newly issued debt are reduced. In this economy, where agents never switch types (borrower/lender), the beneficial debt deflation effect dominates the debt limit effect, such that borrowers are better off with higher inflation rates. In contrast, if the borrowing limit were exogenously tightened, say, by an exogenous reduction of the fraction of seizable income, borrowers' welfare would tend to decrease. The apparent reason is that this impulse lacks the beneficial effect from a reduction of the initial debt burden, while it reduces initial and future consumption possibilities of borrowers due to a tighter effective debt limit.

For the main part of our analysis, we consider an incomplete market model economy with idiosyncratic risk (see Huggett, 1993). Agents differ with regard to their random

[9]Notably, this pecuniary externality (non-trivially) applies also for the opposite case of lower inflation, which increases the maximum debt repayment: Given that agents do not internalize how their demand for funds affects the real interest rate, a lower inflation rate can cause an increase in debt due to borrowers exploiting the higher debt limit, which tends to increase the real cost of borrowing. A related externality with regard to the real interest rate is discussed by Smith (2009).

individual income, while they are equally impatient.[10] When an agent draws a very low realization of income, he is willing to borrow up to the debt limit. The adverse (beneficial) effect of a higher (lower) inflation rate that tends to lower (raise) the effective debt limit and, correspondingly, the maximum amount of borrowed funds at issuance might then outweigh the beneficial (adverse) debt deflation effect. This is actually the case when the probability of drawing again a low realization of individual income at maturity is small enough, such that the marginal valuation of funds at issuance is sufficiently higher than the expected marginal valuation of funds at maturity. Ex-ante, a borrower tends to prefer a lower inflation rate and thus a higher effective debt limit even with the higher debt repayment, if he has a relatively high valuation of funds when debt is issued.[11]

We examine two versions of the incomplete market model with idiosyncratic risk. For the first version, we assume that preferences are linear-quadratic and that income shocks ensure that the borrowing constraint always binds for borrowers, facilitating aggregation and allowing for the derivation of analytical results. Under these assumptions, the competitive equilibrium of the heterogenous agents economy can be characterized in terms of a representative borrower and a representative lender. For this economy, we show analytically that a reduction in the inflation rate can enhance welfare of the representative borrower if the autocorrelation of income shocks is sufficiently low, which tends to raise the gain from the debt limit effect relative to the debt deflation effect. The reason is that a constrained borrower is under a lower autocorrelation of individual income less likely to be constrained at maturity, such that the expected marginal utility of consumption at maturity is lower than the marginal utility of consumption at issuance. With this favorable effect for constrained agents, monetary policy can in principle enhance aggregate welfare by lowering inflation.

To quantitatively assess the effects of changes in the inflation rate, we apply a second

[10] This set-up closely relates to Auclert's (2017) incomplete market model, which he uses to examine redistribution of monetary policy. In contrast to our model, the borrowing constraint in his model limits issued debt rather than outstanding debt, such that the changes in inflation does not alter the effective debt limit.

[11] Studies on monetary policy in incomplete market economies with zero debt and fixed borrowing limits typically find effects of higher inflation rates that are beneficial for borrowers (see e.g. Akyol, 2004, Algan and Ragot, 2010, or Kryvtsov et al. 2011).

version of the model, imposing less restrictive assumptions. Specifically, we consider a standard CRRA utility function, long-term debt, and a more realistic income process, such that the borrowing constraint is not permanently binding. Given that this version cannot be solved analytically, we calibrate the model to match characteristics of US postwar data and solve it numerically. The calibration is based on an inflation rate of 2%. We then assume that the central bank reduces the average inflation rate to -2%. We find that borrowers with a high initial debt position suffer most from lower inflation, given that the debt deflation effect is dominant for them. In contrast, borrowers who are initially less indebted gain from lower inflation due to a dominant debt limit effect. Apparently, a household with positive wealth benefits from both effects when the central bank reduces the inflation rate: Initial real wealth increases as well as debt limits in future periods in which they might be constrained. For our benchmark calibration, we find that aggregate welfare losses due to the inflation reduction via the (conventional) effects of initial debt deflation are reduced by 83% via the effects induced by the borrowing constraint.[12]

To assess the sensitivity of these results, we vary the maturity of debt, examine an equally-sized increase (instead of a reduction) in inflation, and we re-calibrate the model for an alternative income process with lower autocorrelation. Firstly, a reduction of debt maturity leads to an almost proportional reduction of the welfare effects. As described by Doepke and Schneider (2006), debt deflation effects of non-transitory inflation changes increase with the maturity of nominal debt. Likewise, fixing nominal payments for longer terms is crucial for the effects of monetary policy via nominal rigidities induced by fixed-rate mortgage contracts (see Gariga et al., 2017). The debt limit effect is also enhanced with higher maturities, which – like a lower autocorrelation of income – increase the likelihood that borrowers are unconstrained at maturity. Secondly, we find that an increase in inflation leads to almost symmetric effects compared to an equally-sized inflation reduction. These effects are slightly less pronounced, given that the distortionary effects of the borrowing constraint are reduced under higher inflation rates. Finally, we also consider

[12] As a measure for aggregate welfare, we apply agents' ex-ante expected lifetime utility, which relates to an utilitarian welfare measure.

a lower autocorrelation for the income process, as suggested by Guvenen (2007) for the US and by Floden and Line (2001) for Sweden. Re-calibrating the model for Guvenen's (2007) estimates, we find that aggregate welfare (slightly) increases for a reduction in the inflation rate, consistent with the analytical results derived for the simplified version of the model.

In Section 2, we examine the redistributive effects of monetary policy in a stylized model with two agents which are characterized by different degrees of impatience. In Section 3, we apply a model where heterogeneity of agents, instead, originates from idiosyncratic income shocks, and examine the inflation effects analytically as well as numerically. Section 4 concludes.

2 A model with patient and impatient agents

Before we examine financial frictions for monetary policy effects in a Huggett (1993) type model (see Section 3), we analyze the effects in a more stylized model. We assume that two types of agents differ with regard to their degree of patience induced by different discount factors (as in Kiyotaki and Moore, 1997). The patient agents with the higher discount factor will permanently be lenders and the impatient agents with the lower discount factor will permanently be borrowers. The persistence of agents' types will be the main difference between this model and the model in Section 3, where agents might switch roles in the credit market depending on their particular income draws and their endogenous wealth positions.

2.1 The set-up

There is a continuum of infinitely lived agents of mass two, who have equal income from an exogenous labor supply, consume and trade one-period nominal non-state contingent discount bonds at the issuance price $1/R_t$ $(= Q_t)$, paying one unit of currency in period t. For simplicity, we neglect uncertainty and disregard holdings of fiat money, which can be interpreted as the limit case of a cashless economy, while we assume that money only serves as a unit of account (see also Sheedy, 2014, or Auclert, 2019). Households maximize the present value of utilities $\sum_{t=0}^{\infty}(\beta_i)^t u(c_{i,t})$ where $c_{i,t}$ is consumption of agent i and $i = l$ $(i = b)$ is the index of lenders (borrower), who constitute half of the population.

The parameter β_i is the discount factor of agent i and satisfies $\beta_b < \beta_l < 1$. The utility function is identical for all agents and satisfies $u' > 0$ and $u'' < 0$. Agent i's budget constraint in nominal terms is given by

$$P_t c_{i,t} = -(S_{i,t+1}/R_t) + S_{i,t} + P_t y_{i,t}, \tag{1}$$

where P_t denotes the price level, $S_{i,t}$ denotes nominal debt with $S_{l,t} > 0$ and $S_{b,t} < 0$. The endowment $y_{i,t}$ will be identical for all agents, $y_{i,t} = y_t$. In each period, agents first trade in the asset market before they enter the goods market.

As the central element of our analysis, we consider that debt is restricted by current income, for which several studies found empirical support.[13] To rationalize this observation, we consider that agents cannot commit to repay debt. We assume that debt can be renegotiated after issuance. Borrowers might then make a take-it-of-leave-it offer to reduce the value of outstanding debt. Lenders who reject this offer, can take borrowers to court and can seize their available income up to a fraction $\gamma < 1$ (due to imperfections in legal enforcement). Hence, a repudiation-proof debt contract restricts debt repayment to $\gamma P_t y_{i,t}$, leading to the following borrowing constraint[14]

$$-S_{i,t+1} \leq \gamma P_t y_{i,t}. \tag{2}$$

In real terms, i.e. in terms of period t commodities, the budget and borrowing constraints are given by $c_{i,t} = -s_{i,t+1}/R_t + s_{i,t}/\pi_t + y_t$ and $-s_{i,t+1} \leq \gamma y_{i,t}$, where $\pi_t := P_t/P_{t-1}$ denotes the inflation rate and $s_{i,t+1} := S_{i,t+1}/P_t$ the real value of wealth at the end of the period t, which is a predetermined state variable in $t+1$. Accordingly, real end-of-period debt $s_{i,t+1}$ is constrained by a fraction of real income in period t, $y_{i,t}$. Yet, when debt matures, prices might have changed, such that the real value $s_{i,t+1}$ has to be adjusted by the inflation rate to account for real debt burden in terms of commodities at maturity, i.e. $s_{i,t+1}/\pi_{t+1} = S_{i,t+1}/P_{t+1}$. Accordingly, the borrowing constraint $-S_{i,t+1}/P_{t+1} \leq \gamma y_{i,t}/\pi_{t+1}$ shows that

[13] Examples are Japelli and Pagano (1989), Jappelli (1990), Duca and Rosenthal (1993), Del Río and Young (2006), Choi et al. (2018), Dettling and Hsu (2018). Lian and Ma (2019) and Drechsel (2019) further provide evidence that firms' borrowing is constrained by their earnings.

[14] Studies where borrowing is also constrained by the current value of income, are for example Laibson et al. (2003), Mendoza (2006), Bianchi (2011), Benigno et al., (2016), Korinek (2018), Schmitt-Grohe and Uribe (2019).

a higher inflation rate reduces the limit for debt repayment in terms of commodities at maturity $t+1$. Maximizing lifetime utility subject to the budget- and borrowing constraints, leads to the borrowers' and lenders' first order conditions given by

$$u'(c_{b,t})/R_t = \beta_b u'(c_{b,t+1})\pi_{t+1}^{-1} + \zeta_{b,t}, \tag{3}$$

$$u'(c_{l,t})/R_t = \beta_l u'(c_{l,t+1})\pi_{t+1}^{-1}, \tag{4}$$

where $\zeta_{b,t}$ denotes the multiplier on the borrowing constraint (2), which is irrelevant for lenders. Further, the associated complementary slackness condition, $\zeta_{b,t}(\gamma y_{i,t}+s_{b,t+1}) \geq 0$, holds.

In this cashless economy, the central bank can control the nominal interest rate via a channel system. Given that changes in the nominal interest rate will affect the (expected) inflation rate, we will assume, for convenience, that the central bank controls the inflation rate by setting the interest rate in order to meet specific inflation targets, as for example in Sheedy (2014). Given that there is no aggregate uncertainty, we will focus on constant inflation targets, $\pi > 0$. Notably, the inflation choice might imply values for the nominal interest rate for which the zero lower bound, $R_t \geq 1$, is binding.

The equilibrium is then a set of sequences $\left\{c_{b,t}, c_{l,t}, s_{b,t+1}, s_{l,t+1}, R_t, \zeta_{b,t} \geq 0\right\}_{t=0}^{\infty}$ for a given constant inflation rate $\pi > 0$ and a given constant endowment $y_{i,t} = y > 0$ satisfying (3) and (4), $c_{b,t} = -(s_{b,t+1}/R_t) + (s_{b,t}/\pi) + y$, $c_{b,t} + c_{l,t} = 2y$, $-s_{b,t+1} \leq \gamma y$, $\zeta_{b,t}(\gamma y_{i,t} + s_{b,t+1}) \geq 0$, and $-s_{b,t} = s_{l,t}$, given $-s_{b,0} = s_{l,0}$. The real interest rate satisfies (4) and will be strictly positive in a long-run equilibrium, i.e. it equals the inverse of the lenders' discount factor, $R/\pi = 1/\beta_l > 1$. Given that $\beta_b < \beta_l$, borrowers will be constrained in a long-run equilibrium, $\zeta_b > 0$ (see 3).

2.2 Results

We now examine the effects of a permanent change in the inflation rate in this simple economy. Specifically, we consider an unanticipated permanent inflation shock in period $t = 0$, where borrowers are endowed with beginning-of-period wealth $s_{b,0} = S_{b,0}/P_{-1}$. Suppose that the latter is sufficiently close to its steady state value, such that the economy will be in the steady state in period $t \geq 1$. Using the steady state real interest rate,

$R/\pi = 1/\beta_l$, and that borrowers are always constrained, $s_{b,t} = -\gamma y$, the borrowers' budget constraint, $c_{b,t} = -s_{b,t+1}R_t^{-1} + s_{b,t}\pi_t^{-1} + y$, implies initial consumption and steady state consumption (in $t \geq 1$) to satisfy

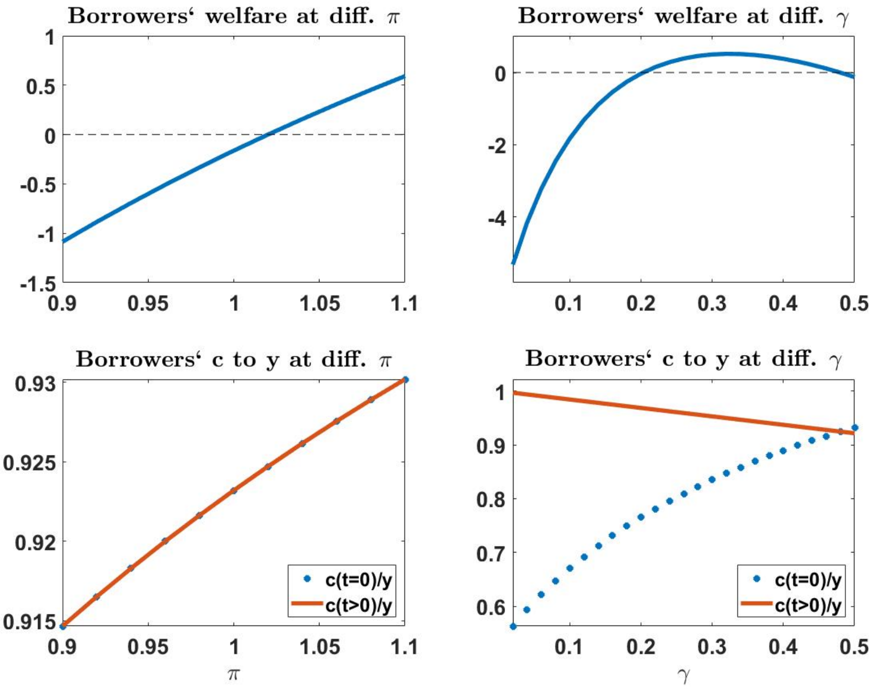
$$c_{b,0} = \underbrace{[s_{b,0}/\pi]}_{A.)\text{ initial debt deflation effect}} + \underbrace{\gamma\,[y/R_0(c_{l,0}, c_{l,1}, \pi)]}_{B.)\text{ initial debt limit effect}} + y, \tag{5}$$

$$c_{b,t} = \underbrace{-\underbrace{[\gamma y/\pi]}_{C.)\text{ debt deflation effect}} + \underbrace{[\gamma y/\pi]\beta_l}_{D.)\text{ debt limit effect}}}_{<0} + y, \ \forall t \geq 1, \tag{6}$$

where the beginning of period stock of debt, $s_{b,0} < 0$, is given. Consider an unexpected permanent increase in the inflation rate in period 0. This tends to increase borrowers' consumption in period 0 according to the *initial debt deflation effect* (see A. in 5), which is independent of the borrowing constraint. At the same time, higher inflation tends to reduce $c_{b,0}$ due to the *initial debt limit effect* (see B.). Specifically, a higher inflation rate causes lenders' to demand a higher nominal interest rate according to their credit supply schedule (4), such that the amount of funds raised at issuance $\gamma y/R_0$ decreases. From $t = 1$ onwards, the economy is in the steady state, where the real interest satisfies $R/\pi = 1/\beta_l > 1$. As in $t = 0$, the *debt deflation effect* (C.) tends to raise and the *debt limit effect* (D.) tends to reduce borrowers' consumption under higher inflation (see 6). In contrast to the initial period, the latter effect is unambiguously weaker than the former, as debt is rolled over at a constant positive interest rate. Hence, borrowers' consumption strictly increases with higher inflation for $t \geq 1$. Two aspects should be noted here.

Firstly, a higher inflation rate would have no further effect on borrowers' consumption than the initial debt deflation effect (A.), if the borrowing limit were specified terms of commodities in period $t+1$. If borrowing were instead limited by $-S_{i,t+1} \leq \gamma P_{t+1}y_t \Longleftrightarrow -s_{i,t+1}/\pi_{t+1} \leq \gamma y_t$, borrowers' consumption in $t \geq 1$ (in which the economy is in a steady state) would be given by $c_b = -\gamma y\,(1 - \beta_l) + y$. Monetary policy would then be neutral in the steady state, since the debt limit is not affected by price changes. Secondly, a permanent reduction of the fraction of seizable income γ starting in period $t = 0$, which for example might be imposed by regulation, is actually not equivalent to an increase

Figure 1: Welfare (relative to a reference economy with $\pi = 1.02$ and $\gamma = 0.487$, in percent) and consumption share of relatively impatient agents



in the inflation, which can be seen from (5). The reason is that a change of γ in $t = 0$ cannot affect real initial wealth $s_{b,0}$ (for which γ would already have to be changed in $t = -1$).

For demonstrative purposes, we provide quantitative results for the effects of inflation. To abstract from transitional dynamics, we assume that borrowers are initially endowed with the steady state stock of debt, $s_{b,0} = -\gamma y$. Notably, the latter assumption implies that the borrower will be in a steady state in all periods $t \geq 0$ with $s_{b,t} = -\gamma y$ regardless of the inflation rate. Figure 1 shows the effects of the inflation rate π and the parameter γ on borrower's consumption share $c_{b,t}/y$ and welfare for the period $t = 0$ and $t > 0$. The corresponding effects on lenders are shown in Figure 9 in Appendix C.

We compute welfare of borrowers by $v_b = \sum_{t=0}^{\infty} \beta_b^t u(c_{b,t}) = u(-\gamma(y/\pi)(1-\beta_l) + y)/(1-\beta_b)$ and display consumption equivalents $CE_b = u^{-1}((1-\beta_b)v_b)$ relative to a reference economy with 2% inflation. The chosen benchmark parameter values are $y = 0.56$, $\beta_l = 0.82$, $\beta_l = 0.84$ and $\gamma = 0.487$ with a CRRA utility function $u(c_i) = c_i^{1-\sigma}/(1-\sigma)$ and $\sigma = 2$ (see Section 3.3 for a discussion of the parameter values). The first column shows the effects of a change in the inflation rate. The consumption share $c_{b,t}/y$ and welfare of borrowers unambiguously increase with the inflation rate, in accordance with the effects described above. The second column of Figure 1 displays the effects of changes in the fraction γ at a constant inflation rate π. A reduction of γ has a positive impact on borrowers' consumption share in $t > 0$ by lowering debt (see solid line), which qualitatively accords to the impact of a higher inflation rate. In contrast to the latter, a lower value γ has an adverse effect on borrowers' initial consumption share, since it simply implies a more restricted access to external funds under a given initial debt level $s_{b,0}$ (see 5). For $\gamma < 0.32$, borrowers' welfare monotonically decreases with a tighter borrowing constraint induced by a lower fraction γ. For larger values of γ that are nevertheless associated with a binding borrowing constraint, we find that borrowers' welfare can increase under a reduction of γ. The reason is that adverse effects of an increased borrowing on (higher) interest rate and future consumption possibilities are not internalized by agents (see Gottardi and Kuebler, 2015, for a similar finding).

3 A model with idiosyncratic risk

In this Section, we examine the effects of inflation in a Hugget-type model, where agents have the identical discount factor. Idiosyncratic endowment shocks induce agents to borrow/lend, while there is no aggregate risk. As in the model presented in the previous section, only non-state-contingent nominal debt is available such that agents cannot share risk. This model can in general not be solved analytically, given that agents might have different histories of $y_{i,t}$-draws and their decisions depend on their beginning-of-period wealth $s_{i,t}$. We will therefore apply some simplifying assumptions in the first part of the analysis. Specifically, we consider a constant borrowing limit, a linear-quadratic utility function and we assume that the borrowing constraint binds for agents who draw a low income level, which facilitates aggregation and derivation of analytical results. In the second part, we calibrate a more realistic version of the model to assess the inflation effects in a quantitative way.

3.1 The set-up

Consider an economy with infinitely lived and infinitely many households i of mass two. These households share the same utility function, but might differ with regard to a random idiosyncratic income. Preferences of a household i are given by

$$E_i \sum_{t=0}^{\infty} \beta^t u(c_{i,t}), \tag{7}$$

where E_i denotes an expectations operator and $c_{i,t}$ consumption of household i. As before, the utility function $u(c_{i,t})$ is assumed to satisfy $u' > 0$ and $u'' < 0$. Note that in the subsequent analysis, we will examine the model for two different types of utility functions. First, we apply a linear-quadratic (LQ) utility function, which facilitates aggregation and the derivation of analytical results. Second, we use a standard CRRA utility function for a numerical analysis.

Real income $y_{i,t} = Y_{i,t}/P_t$ is identically and independently distributed over all households, but might be serially correlated over time. We consider a finite set of n possible realizations of the random variable y, y_1,... y_n, where $y_i < y_{i+1}$ and with transition probabilities $p_{k,l}$ from state k to state l and a positive unconditional mean $Ey_i = \overline{y} > 0$. Households who draw an income y_i tend to borrow from households who draw $y_j > y_i$. Shocks are realized at the beginning of each period, before the asset market opens. Once, these shocks are realized, households enter the asset market where they repay debt and can borrow/lend funds from/to other households.

To allow for a more realistic debt maturity, we introduce long-term debt contracts that mature probabilistically (see for example Chatterjee and Eyigungor, 2012). We assume that each unit of outstanding debt matures in the subsequent period with a constant probability θ. Given that a unit bond issued in period $t-k$ leads to the same payoff as an unit bond issued in $t-k'$ with $k' > k > 1$, it is sufficient to keep track of the total number of bonds. Bond units are infinitesimally small, such that for s_{t+1} bond units outstanding at the beginning of period $t+1$ real payment obligations are $\theta s_{t+1}\pi_{t+1}^{-1}$ with certainty. Let Q_t be the issuance price of a unit bond in period t. The budget constraint for a household i in income state $y_{i,t}$ for $i = 1, ..., n$ and wealth state $s_{i,t}$ is

$$P_t c_{i,t} + Q_t \left(S_{i,t+1} - (1-\theta)S_{i,t}\right) = \theta S_{i,t} + P_t y_{i,t}. \tag{8}$$

For one-period debt, $\theta = 1$, the budget constraint reduces to (1), where $R_t = 1/Q_t$. As borrowers cannot commit to repay, the borrowing constraint restricts total outstanding debt $-S_{i,t+1}$ (irrespective of maturity) to a fraction of current income (2). To disclose the main mechanism, we will further apply a simplified borrowing constraint for the derivation of analytical results in the first part of the analysis: $-s_{i,t+1} \leq b$, where the constant b can be interpreted as referring to mean income, $b = \gamma\overline{y}$ (see Section 3.2).

Households aim at maximizing lifetime utility (7) subject to (2) and (8) taking prices as given. The first order conditions for a household i in income state $y_{i,t} = y_j$ for $j = 1, ..., n$ and wealth state $s_{i,t} = s_t$ is

$$u'_{i,t} Q_t = \beta E_{i,t}\left[\left(Q_{t+1}(1-\theta) + \theta\right) u'_{i,t+1}/\pi_{t+1}\right] + \zeta_{i,t}, \tag{9}$$

where $\zeta_{i,t} \geq 0$ denotes the multiplier on (2). Further, the budget constraint (8) is binding and the complementary slackness conditions for (2), $0 = \zeta_{i,t}(\gamma y_j + s_{i,t+1})$, and $\zeta_{i,t} \geq 0$, hold. Notably, the first order condition (9) for one-period debt ($\theta = 1$) simplifies to $u'_{i,t}/R_t = \beta E_{i,t}[u'_{i,t+1}\pi_{t+1}^{-1}] + \zeta_{i,t}$.

In equilibrium, prices adjust such that plans are realized and markets clear. A competitive equilibrium is a set of sequences $\{c_{i,t}, s_{i,t+1}, Q_t, \zeta_{i,t}\}_{t=0}^{\infty}$ satisfying (9), $-s_{t+1} \leq \gamma y_j$, $c_t + Q_t\left(s_{t+1} - (1-\theta)s_t\pi_t^{-1}\right) = \theta s_t \pi_t^{-1} + y_{i,t}$, $y_t = \Sigma_i y_{i,t} = \Sigma_i c_{i,t}$ and $\Sigma_i s_{i,t+1} = 0$, and the complementary slackness conditions for a given inflation rate π_t and given $s_{i,0}$. The first best allocation $\{c^*_{i,t}\}_{t=0}^{\infty}$ evidently satisfies $u'(c^*_{i,t}) = u'(c^*_{j,t})$ for all agents $i \neq j$, which we will consider as a benchmark case.

3.2 A version with two representative agents

In this subsection, we apply a simple version the model and analytically examine the main effects of changes in the inflation rate. We consider two realizations for income, y_1 and y_2, with symmetric transition probabilities, and we consider one-period debt, $\theta = 1$. To derive analytical results, we further impose a linear-quadratic utility function.

Assumption 1 *Households' preferences satisfy* $u(c_{i,t}) = (\delta c_{i,t} - c_{i,t}^2)$*, where* $\delta \geq \Sigma_i y_i$*.*

When preferences satisfy Assumption 1, the marginal utilities are linear in individual consumption, which greatly facilitate aggregation over individual household choices. We further consider a constant borrowing limit b and restrict our attention to the case where the variance of the preference shocks is sufficiently large such that the borrowing constraint will always be binding for agents drawing y_1. To achieve this, we apply a relatively large income difference $y_2 - y_1$ compared to the parameter b governing the tightness of the borrowing constraint.

Assumption 2 *The borrowing constraint is given by* $-s_{i,t+1} \leq b$*. Idiosyncratic income satisfies* $y_{i,t} \in \{y_2, y_1\}$*, where* $p_{12} = p_{21}$*,* $p_{11} = p_{22} > 0$*, and* $(y_2 - y_1)/b$ *is sufficiently large such that* $\zeta_{j,t} > 0$ *for all households* j *drawing* y_1*.*

Hence, borrowers' end-of-period wealth positions equals $-b$. Accordingly, lenders, which are of the same mass as borrowers, have a wealth position equal to (minus) the debt level of borrowers (b). As for the model with different degrees of patience, we analyze the effects of inflation on agents initially endowed with $s_{i,0} = -b$ or $s_{i,0} = b$ and $\Sigma_i s_{i,0} = 0$ to abstract from transitional dynamics. Under Assumptions 1 and 2, we can analytically aggregate over individual choices of agents. We separately analyze two types of agents, borrowers drawing y_1 and potential lenders drawing y_2. The choices of the former are characterized by the conditions $(\delta - 2c_{(b,i),t})/R_t = (\beta/\pi)\left[p_{11}(\delta - 2c_{(b,i),t+1}) + p_{12}(\delta - 2c_{(l,i),t+1})\right] + \zeta_{(b,i),t}$, $-s_{(b,i),t+1} \leq b$, and $c_{(b,i),t} = -s_{(b,i),t+1}R_t^{-1} + s_{(b,i),t}\pi^{-1} + y_1$, where $\zeta_{(b,i),t} \geq 0$ and $\zeta_{(b,i),t}\left(s_{(b,i),t+1} + b\right) = 0$. Given that all conditions are linear in the choice variables for $\zeta_{(b,i),t} > 0$, we can easily aggregate. Let $c_{b,t} = \Sigma_{b,i} c_{(b,i),t}$, $\zeta_{b,t} = \Sigma_{b,i}\zeta_{(b,i),t}$ and $s_{b,t+1} = \Sigma_{b,i} s_{(b,i),t+1}$. Then, we get the following set of conditions describing the behavior of a representative borrower:

$$(\delta - 2c_{b,t})/R_t = (\beta/\pi)\left[p_{11}(\delta - 2c_{b,t+1}) + p_{12}(\delta - 2c_{l,t+1})\right] + \zeta_{b,t}, \tag{10}$$

$$-s_{b,t+1} = b, \tag{11}$$

$$c_{b,t} = -(s_{b,t+1}/R_t) - p_{11}(b/\pi) + p_{21}(b/\pi) + y_1, \tag{12}$$

and $\zeta_{b,t} > 0$. Note that we used that beginning of period wealth either equals b or $-b$, depending on whether the current borrower was a lender or a borrower in the previous period. Using the law of large numbers, a fraction of p_{11} (p_{21}) of previous borrowers (lenders) draw y_1 in the current period. Thus, current period initial wealth of the representative borrower equals the weighted average $-p_{11}(b/\pi) + p_{21}(b/\pi)$. Apparently, the same arguments apply for all agents drawing y_2, such that we can proceed analogously and get the following conditions describing the behavior of a representative lender:

$$(\delta - 2c_{l,t})/R_t = (\beta/\pi)\left[p_{21}(\delta - 2c_{b,t+1}) + p_{22}(\delta - 2c_{l,t+1})\right], \tag{13}$$

$$c_{l,t} = -(s_{l,t+1}/R_t) - p_{12}(b/\pi) + p_{22}(b/\pi) + y_2. \tag{14}$$

Hence, we can characterize a competitive equilibrium in terms of a representative borrower and lender.

Proposition 1 *Under Assumptions 1 and 2, a competitive equilibrium with one-period debt ($\theta = 1$) can be characterized as a set of sequences $\{c_{b,t},\ c_{l,t},\ s_{b,t+1},\ R_t,\ \zeta_{b,t} > 0\}_{t=0}^{\infty}$ satisfying (10), (11), (13),*

$$c_{b,t} - c_{l,t} = -(2s_{b,t+1}/R_t) - (p_{11} - p_{21})(b/\pi) + (p_{12} - p_{22})(b/\pi) + y_1 - y_2, \tag{15}$$
$$c_{b,t} + c_{l,t} = y_1 + y_2, \tag{16}$$

for a given inflation rate $\pi > 0$.

We now examine how monetary policy affects the allocation and aggregate welfare in the representative agents economy given in Proposition 1. Specifically, we analyze the effects of the inflation rate on borrowers' consumption and on aggregate welfare, measured as ex-ante expected lifetime utility. This can be interpreted as measuring welfare of an unborn agent facing the equally likely possibility of being a representative borrower or a representative lender, which is equivalent to a utilitarian welfare measure. Under Assumption 2, (10) and (13) imply that consumption of the representative borrower is strictly smaller than consumption of the representative lender due to the binding borrowing constraint. Combining (10) and (13) and using $\zeta_{b,t} > 0$, shows that the marginal utility of the representative borrower is strictly larger than the marginal utility of the representative lender, $(\delta - 2c_{b,t}) > (\delta - 2c_{l,t})$. As long as this inequality holds, a redistribution of consumption from the latter to the former increases aggregate welfare. It can be shown that this can be induced by reducing the inflation rate if the serial correlation of endowment shocks is not too high. Then, monetary policy can, in principle, also implement first best as long as the zero lower bound is respected.

Proposition 2 *Consider a competitive equilibrium as given in Proposition 1. A reduction in the inflation rate raises borrowers' consumption and enhances aggregate welfare if* $p_{12} > (1 - \beta)/2$*. Monetary policy is then able to implement first best if* $1 \leq 2b[1 + (p_{21} - p_{11})/\beta]/(y_2 - y_1)$*.*

Proof. See Appendix A. ■

According to Proposition 2, monetary policy should choose a low inflation rate to maximize aggregate welfare if the probability of changing income types is sufficiently large, i.e. $p_{21} = p_{12} > (1 - \beta)/2$. The reason for this result is that inflation exerts the previously discussed opposing effects, i.e. debt deflation and debt limit (see Section 2), on borrowers. Under the borrowing constraint, $-s_b \leq b$ (as assumed here), the amount of funds that can be issued b/R and the repayment b/π decrease with the inflation rate. Thus, monetary policy is non-neutral, while its overall impact on borrowers depends on the subjective valuation of funds at issuance and at maturity, which depends on the marginal utility of consumption.

Consider for example a household who draws y_1 today and borrows funds up to the borrowing constraint. If the probability of being unconstrained at maturity (for y_2) is positive, its expected marginal utility then tends to be lower than today. This household would gain from a proportional reduction in the nominal interest rate, which raises the amount of funds that can be borrowed today, even if its is accompanied by a proportional reduction in the inflation rate, which tends to raise real debt repayment. Thus, under a sufficiently large probability of drawing a high income shock and being unconstrained at maturity the debt limit effect dominates the debt deflation effect, such that monetary policy should lower rather than raise inflation to benefit borrowers. This result is consistent with the findings in Section 2, where borrowers are permanently constrained and therefore gain from higher rather than lower inflation. Though, the condition for lower inflation to enhance welfare, i.e. $p_{21} = p_{12} > (1 - \beta)/2$, seems to be fairly week (given that discount factors are typically close to one), it remains to assess whether the arguments made are of quantitative relevance.

3.3 A calibrated version

The previous analysis has shown that monetary policy can enhance aggregate welfare by reducing inflation and the nominal interest rate, when borrowing agents are less likely to be constrained at maturity. Yet, this analysis has been conducted under simplifying assumptions on preferences, the debt limit, maturity, and shocks. Here, we examine a less stylized framework, which will be calibrated for US data. For this, we omit the Assumptions 1 and 2. We apply a conventional CRRA period utility function for households $i \in [0,1]$ $u(c_{i,t}) := c_{i,t}^{1-\sigma}/(1-\sigma)$, where $\sigma > 0$. We further use the borrowing constraint (2) and we do not restrict the analysis to the case where the borrowing constraint is always binding for borrowers. As a consequence, individual wealth/debt of agents can vary over time depending on the individual history of income shocks. The realizations of these shocks are now assumed to satisfy $y_{i,t} \in \{y_1, y_2, ..., y_n\}$, where $0 < y_j < y_{j+1}$ for $j = 1, ..., n-1$, and to follow a first-order Markov process with transition matrix $\mathcal{P}$. The elements are $\mathcal{P}_{k,l} := p_{k,l}$ for $k, l = 1, ..., n$, where $p_{k,l}$ is the probability to switch from state k in $t-1$ to state l in period t.

We examine the effects of the following policy experiment. Initially, the economy is in the stationary equilibrium induced by the benchmark inflation rate of 2%. We then introduce an unexpected permanent reduction in the inflation rate to -2% in period 0 and assess the effects on the allocation and agents' welfare. After the change in inflation, the economy leaves the stationary equilibrium induced by an inflation rate of 2% and converges to the new one under the lower rate of -2%. Therefore, we first calculate the stationary equilibrium for both inflation rates and then the transition path from the old to the new stationary equilibrium.

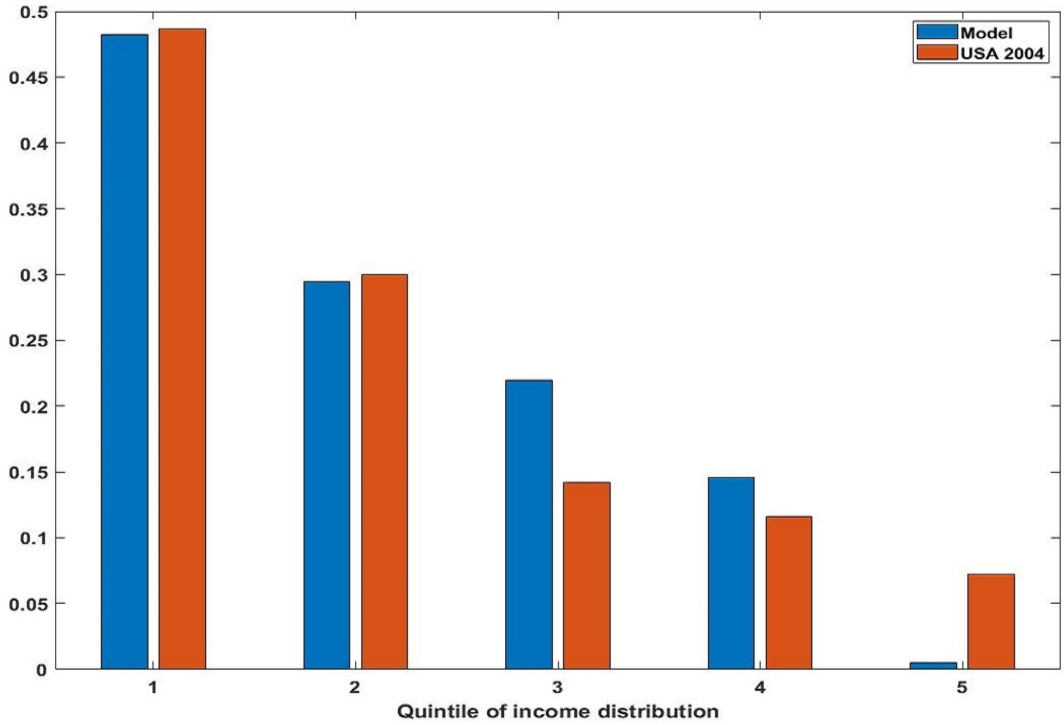
Let λ be a distribution of agents, where $\lambda(s, y)$ is the measure of agents with wealth s and income y. The stationary equilibrium then consists of a price Q, constant policy functions $c(s, y)$ and $s'(s, y)$ and a distribution $\lambda(s, y)$ consistent with a particular inflation rate such that 1) decision rules solve the individual optimization problem, 2) markets clear $\sum_{s,y} \lambda(s, y)c(s, y) = \sum_{s,y} \lambda(s, y)y$ and $\sum_{s,y} \lambda(s, y)s'(s, y) = 0$, and 3) $\lambda(s, y)$ is time invariant (see Appendix B.3). Having constructed the stationary equilibria, we calculate the transition path from the old to the new stationary equilibrium (see Appendix B.4).

Note that the policy functions, wealth distribution and nominal interest rate are not constant over the transition period, but then converge to the time invariant functions and values of the stationary equilibrium under the new inflation rate of -2%.

Calibration To solve the model numerically, we need to assign values for the degree of relative risk aversion σ, the seizable fraction of income γ, debt maturity $1/\theta$, the subjective discount factor β, and the moments of the idiosyncratic income process. The length of a period is assumed to equal 1 year. For σ, we apply the value 2 in accordance with many related studies. As the empirical counterpart of debt, we apply installment loans, where we disregard loans for vehicles and housing. The reason is that the latter typically serve as collateral, while debt is not collateralized in our model. We apply US postwar data for installment loans and after tax income in 2004 taken from the CBO and the Survey of Consumer Finances (see Appendix B.1). Based on these data, we set γ equal to 0.49 to match the ratio of debt to income in the first income quintile, and $1/\theta$ equal to 2 to match the average maturity. While empirical interest rates on installment loans are relatively high, we calibrate the model, i.e. we set $\beta = 0.83$, to get an annual real rate of return $r^*_{t+1} = \left[Q_{t+1}(1-\theta)+\theta\right]/(Q_t\pi)$ of 4%. This value relates to a risk free rate, which is more suited for our model specification, since it does not account for default (risk).

For the income process, we assume that log individual annual income follows an AR1 process, $\ln(y_{i,t}) = \rho\ln(y_{i,t}) + \epsilon_{i,t}$, with $\epsilon_{i,t}$ i.i. normally distributed with mean 0, variance σ^2_ϵ. We apply Tauchen and Hussey's (1991) algorithm for the five states of the log-labour-income process. This leads to the transition matrix $\mathcal{P}$ given in Appendix B.2 and a stationary distribution with 20% of the population in each income state, given by $y_1 = 0.49$, $y_2 = 0.76$, $y_3 = 1$, $y_4 = 1.31$ and $y_5 = 2.04$. For the benchmark parametrization, we use Floden and Linde's (2001) estimates for the autocorrelation coefficient and the variance, $\rho = 0.9136$ and $\sigma^2_\epsilon = 0.0426$. For an alternative specification, we use Guvenen's (2007) income process estimates, providing a lower autocorrelation and a lower variance, $\rho = 0.821$ and $\sigma^2_\epsilon = 0.029$. We compute the solution of the model applying an endogenous grid point method to calculate the stationary equilibrium (see Appendix B.3). Under these parameter values, the nominal interest always satisfies the zero lower bound.

Figure 2: Ratios of debt to income for US income quintiles (model: blue, data: red)



To see how the model implied distribution of debt for a benchmark inflation rate of 2% relates to its empirical counterpart, Figure 2 shows the ratios of debt to income for the five income states and the empirical counterparts of 2004.[15] The model is actually able to fit the ratios of debt to income for the income quintiles reasonably well. Yet, the model underestimates the value for the highest income. The reason is that households in the highest income quintile have a relatively low incentive to borrow in our model, as they tend to save for consumption smoothing.

How does inflation affect agents' choices? We first examine the effects of a change in the inflation rate on consumption and saving/borrowing. Assume that the distribution of predetermined wealth s_0 is initially given by the stationary distribution induced by an inflation rate of 2%. Then, monetary policy unexpectedly decreases the inflation rate to -2% and holds it constant at -2% thereafter. The economy leaves the old stationary

[15] The ratios of debt to income for the different income quintiles are calculated by using average installment loans w.o. vehicle installment loans of households with holdings in income quintiles in 2004 from SCF 2004 (for debt) and average after tax income in income quintiles in 2004 from CBPP (for income).

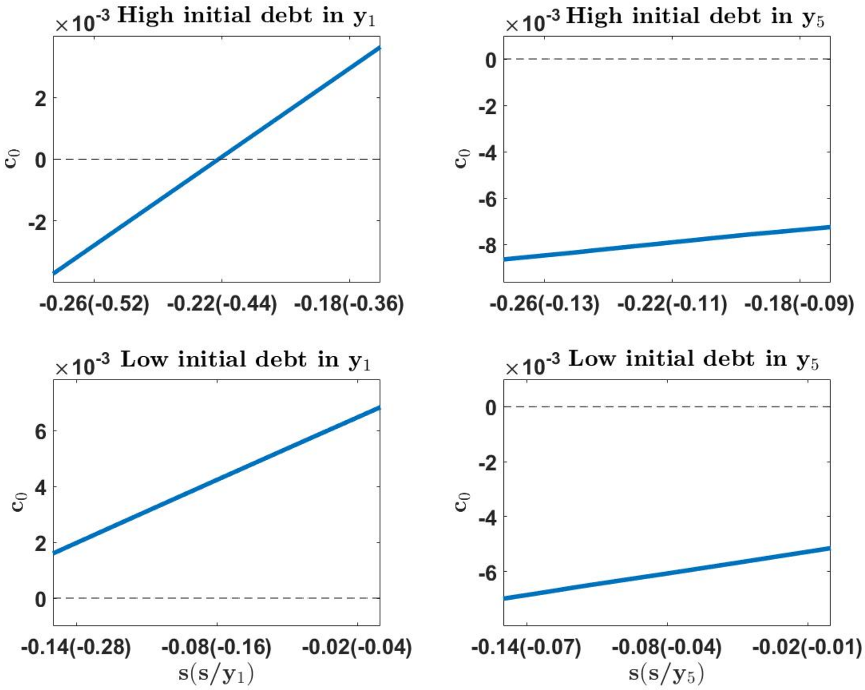
equilibrium under 2% inflation and converges to the new one under -2%.[16] The reduction in inflation has no impact on the distribution of predetermined wealth s_0. Yet, the initial debt deflation effect raises the real value of initial wealth in terms of current period commodities s_0/π. Via the debt deflation effect, the lower inflation rate also tends to raise all future debt repayments in terms of commodities at maturity for non-matured initial debt $(1-\theta)^t s_0/\pi$ and for borrowers who are constrained at issuance. Via the debt limit effect, lower inflation raises the issuance price of debt and the maximum amount of funds that can be borrowed. Whether lower inflation is beneficial or not for a borrower who is constrained at issuance depends – inter alia – on the likelihood to be again constrained at maturity. Remember that the debt deflation effect has dominated the overall welfare result in the model with different degrees of patience, where borrowers are constrained in all periods (see Section 2). In contrast, the debt limit effect can dominate in the economy with idiosyncratic shocks if borrowers who are constrained at issuance have to repay higher debt obligations while being unconstrained at maturity with a positive probability (see Proposition 2).

To unveil the effects on the allocation and on aggregate welfare, we first examine policy functions for the given wealth distribution in period 0. Specifically, we compute the policy functions for consumption $c(s, y)$ and for beginning-of-period wealth $s'(s, y)/\pi$ for different income states of the economy under an inflation rate of 2% and of the economy under a lower inflation rate of -2%. The lower inflation rate reduces the nominal interest rate,[17] implying an increase in the effective limit for borrowed funds. The changes in the period-0 policy functions for consumption and beginning-of-period wealth for a reduction in the inflation rate from 2% to -2% are shown in Figure 3 and 4. For convenience, we focus on the incomes states y_1 and y_5 and on initially indebted agents ($s_0 < 0$), while corresponding policy functions that also include agents with positive initial wealth are shown in Figure 10 in Appendix C.

[16] To calculate the transition path we first compute the old and the new stationary equilibrium (see Appendix B.3). We assume that the economy reaches the new stationary equilibrium after T periods and then calculate the path for the rate of return on debt such that the corresponding policy functions imply a path for the wealth distribution that converges to the wealth distribution of the new stationary equilibrium after T periods (see Appendix B.4).

[17] The net nominal interest rate falls from 6% to 2.115% in period 0 and then converges to 2.125%.

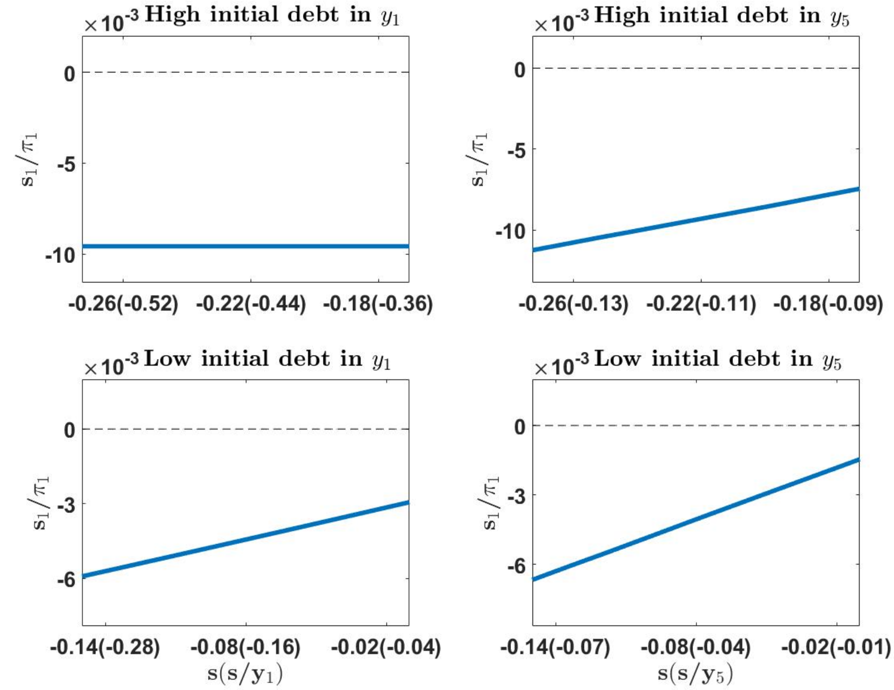
Figure 3: Change in the policy functions for consumption c in period 0 for an inflation reduction from 2% to -2% for distinct ranges of s (and s/y) values



Intuitively, the reduction in the inflation rate increases the effective value of initial debt $-s_0/\pi$ (wealth s_0/π) and thereby tends to decrease (increase) consumption. The changes in the policy functions in Figure 3 show that borrowers in the income state y_1 with relatively high initial debt (see upper left panel) decrease consumption in the initial period due to the debt deflation effect. However, the initial debt deflation effect is not dominant for all initially indebted households. Firstly, constrained borrowers with relatively low initial debt tend to raise consumption by increasing borrowing (i.e., by reducing s'/π, see Figure 4), indicating that the debt limit effect dominates the initial debt deflation effect. Secondly, consumption under low inflation is also higher for unconstrained borrowers with low initial debt in y_1 (see bottom left panel in Figure 3), as these households, who have a relatively high probability to be constrained in future periods, can potentially increase borrowing due to higher effective debt limits in the future. Put differently, their precautionary savings motive is less pronounced due to an improved access to external funds.[18] For the highest income state y_5, for which consumption and

[18] Lower inflation further tends to increase consumption more for positive initial wealth levels s_0 (see

Figure 4: Change in the policy functions for beginning-of-period wealth s'/π in period 0 for an inflation reduction from 2% to -2% for distinct ranges of s (and s/y) values



beginning-of-period wealth are shown in the right hand columns of Figure 3 and 4, borrowers are not constrained and the debt deflation effect dominates, such that they reduce consumption. Finally, it should be noted that the policy functions under a stationary wealth distribution for an inflation rate of -2% are virtually identical with the policy functions in period 0 (see Figure 11 in Appendix C). Hence, the effects for $t = 0$ also apply for the subsequent periods $t \geq 1$.

Who gains from lower inflation? The policy functions presented above have shown changes in consumption and savings due to lower inflation in the initial period in which the shock realizes. To disclose how inflation affects agents' welfare, we calculate the change in expected lifetime utility given by $v(s, y) = E_0 \sum_{t=0}^{\infty} \beta^t c_t(s_t, y_t)^{1-\sigma}/(1-\sigma)$ given $s_0 = s$ and $y_0 = y$. Denote by $v_\pi(s, y)$ the expected lifetime utility of a household with income y and wealth s for a specific inflation rate π. Hence, a reduction in the inflation rate from 2% to -2% increases expected lifetime utility of a household in the initial state (s, y) if $v_{-2}(s, y)-$

first row of Figure 10 in Appendix C).

$v_2(s, y) > 0$. To quantify the welfare consequences of the change in the inflation rate for a household of type (s, y) we express the differences in units of consumption. Therefore, we calculate the percentage change in consumption in the stationary equilibrium with an inflation rate of 2%, in each date and state, for the household of type (s, y) to be indifferent between an inflation rate of 2% and a permanent reduction in the inflation rate to -2%. The gain g of the inflation reduction is then implicitly given by $v_2(s, y; g) = v_{-2}(s, y)$ with $v(s, y; g) = E_0 \sum_{t=0}^{\infty} \beta^t \left((1 + g)c_t(s_t, y_t)\right)^{1-\sigma} / (1 - \sigma)$.

The solid black lines in the left hand column of Figure 5 show the gain $g(s, y)$ for the different income states. Furthermore, the figure splits $g(s, y)$ into the contribution of the effects of initial debt deflation (ID, see dotted lines), which are independent of the borrowing constraint, and of $g(s, y)$ without the effects of initial debt deflation, which captures the monetary non-neutrality due to the borrowing constraint (BC, see dashed line). Notably, effects of initial debt deflation as well as effects due to the borrowing constraint are more persistent under longer-term debt than under one-period debt (see also Figure 6).[19] Let $\widetilde{g}(s, y)$ denote the contribution of $g(s, y)$ without the effects of initial debt deflation, implicitly defined by $v_2(s, y; \widetilde{g}) = v_{-2}(\tilde{s}, y)$ where $\tilde{s}$ is given by $\tilde{s}/0.98 = s/1.02$,[20] such that the effects of initial debt deflation are shut down. The borrowing constraint effects $\widetilde{g}(s, y)$ are then given by the debt limit effects under a lower inflation rate as well as the deflation effects on debt issued in $t \geq 0$,[21] while the contribution of the effects of initial debt deflation are the residual to $g(s, y)$.

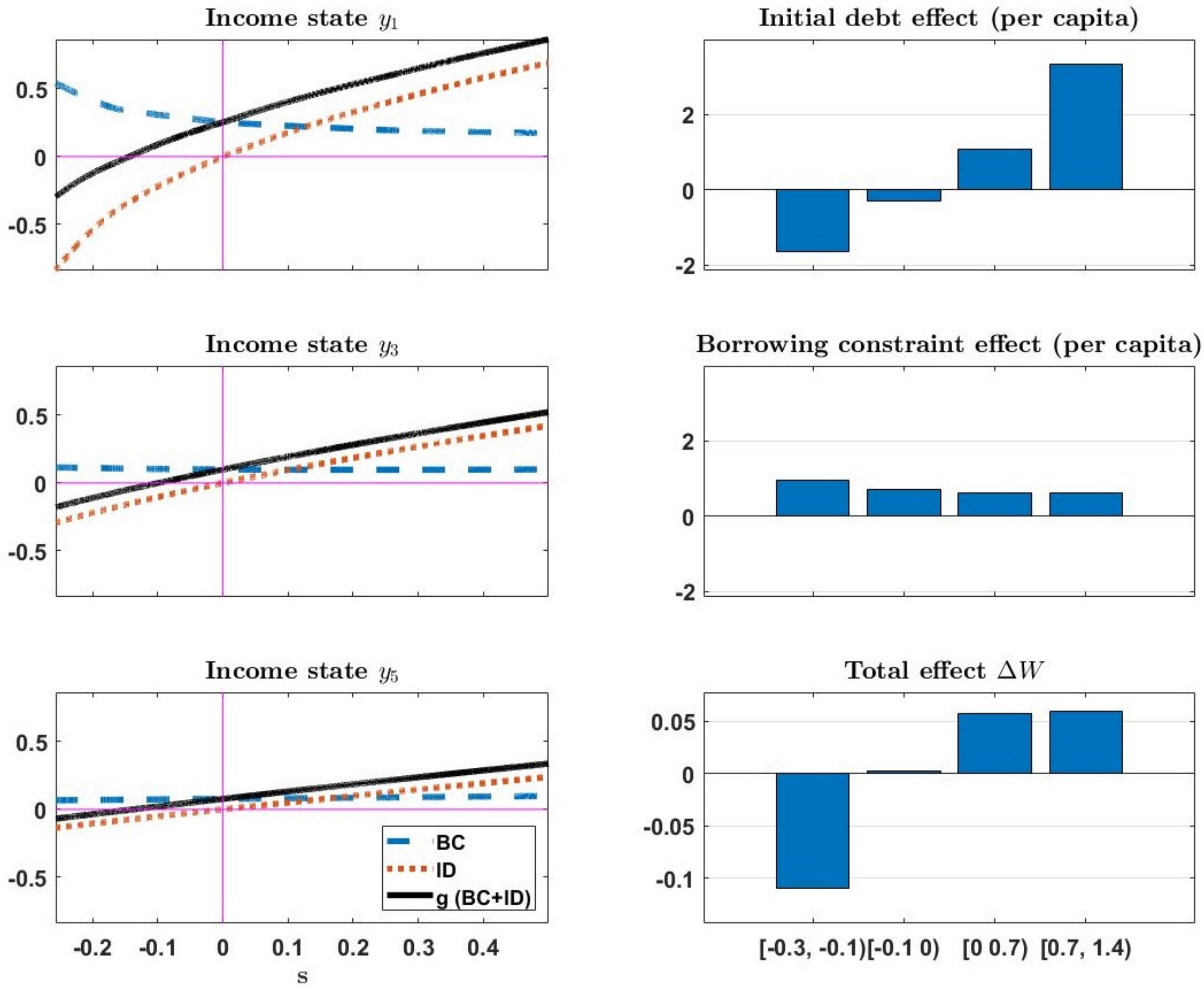
Apparently, the welfare contribution of the effects of initial debt deflation are negative (positive) for households with initial debt (positive wealth). The borrowing constraint effects $\widetilde{g}(s, y)$ tend to increase expected lifetime utility, in particular, of constrained borrowers and households with a high probability to be constrained in future periods by increasing the borrowing limit. However, the borrowing constraint effects tend to in-

[19] For example, the fraction $1 - \theta$ of initial debt that has not matured contributes to the effects of debt deflation in $t = 1$.

[20] Put differently, the effect $v_{-2}(\tilde{s}, y) - v_2(s, y)$ is the difference in expected lifetime utility between a household who lives in an economy with an inflation rate of 2% and has a real value of beginning of period wealth $s/1.02$ and another households who lives in an economy with a permanent reduction in the inflation rate to -2% and has a real value of beginning of period wealth $\tilde{s}/0.98 (= s/1.02)$.

[21] These effects correspond to the effects $B.$), $C.$), and $D.$) in (5) and (6).

Figure 5: Individual welfare effects for three income states and welfare aggregated for four wealth sets [right column; first row: ID (per capita), second row: BC (per capita), third row: ΔW_{s_x}]



crease expected lifetime utility also of wealthier agents due to the increase in the effective debt limit. In total, agents with relatively high initial debt (especially the constrained borrowers) suffer due to dominant effects of initial debt deflation (see also Figure 3). Agents with positive wealth benefit from the reduction in the inflation rate due to both a higher real wealth in the initial period and higher borrowing limits in future periods in which they might be constrained. Importantly, agents with relatively low initial debt, i.e. $s > -0.14$ for y_1, $s > -0.1$ for y_3, and $s > -0.13$ for y_5, also benefit from the lower inflation rate (see Figure 5). This is due to the beneficial debt limit effect which allows to increase borrowing in future periods, where these agents might be constrained. In these cases, the borrowing constraint effects dominate the effects of initial debt deflation.

What are the inflation effects on ex-ante expected lifetime utility? In the previous analysis, we have shown how individual agents' welfare is affected by a reduction in the inflation rate. Here, we assess the effect of inflation on aggregate welfare measured by agents' ex-ante expected lifetime utility. Hence, we examine welfare of agents who are randomly placed into the cross-sectional distribution over individual characteristics in an economy with an inflation rate of either 2% or -2%.[22] As defined above, $v_\pi(s, y)$ is the expected lifetime utility of household of type (s, y) for the inflation rate π and $g(s, y)$ measures by how much this household prefers to be assigned to an economy with an inflation rate of -2% compared to 2% in consumption terms, $g(s, y) = \left(\frac{v_{-2}(s,y)}{v_2(s,y)}\right)^{\frac{1}{1-\sigma}} - 1$. The change in aggregate welfare measured by ex-ante expected lifetime utility is then given by $\Delta W = \left(\frac{\Sigma_{s,y}\lambda_2(s,y)v_{-2}(s,y)}{\Sigma_{s,y}\lambda_2(s,y)v_2(s,y)}\right)^{\frac{1}{1-\sigma}} - 1$, where λ_2 is the wealth distribution before inflation is changed.[23]

The right hand column of Figure 5 shows the welfare effects in percentages of consumption units aggregated over agents within four wealth sets, $s_I \in [-0.3, -0.1)$, $s_{II} \in [-0.1, 0)$, $s_{III} \in [0, 0.7)$, and $s_{IV} \in [0.7, 1.4]$. The right hand panel in the first row displays the *per capita* welfare effects that are solely induced by the effects of initial debt

[22] Given the law of large numbers, such that the probability of drawing a specific individual state equals the mass of agents with this specific individual state, this measure relates to a utilitarian welfare measure.

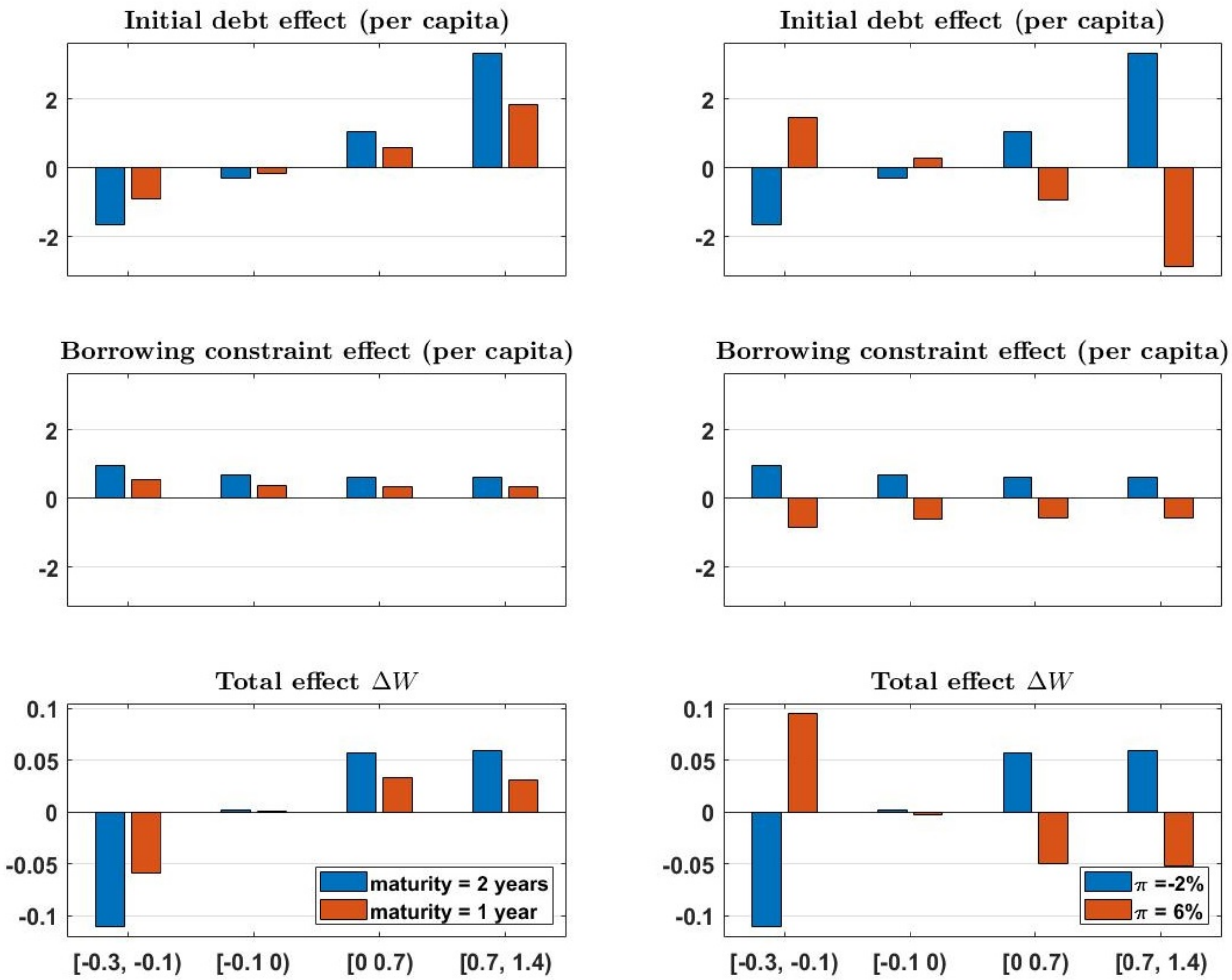
[23] Notably, the distribution of initial real wealth s_0 is not affected by the change in inflation, in contrast to the distribution of real wealth in the subsequent periods.

deflation.[24] Apparently, agents with a high debt position suffer more from a reduction in the inflation rate, while agents with positive savings gain from the lower inflation rate. The right hand panel in the second row shows that welfare within all wealth sets is positively affected by the borrowing constraint effects. The increased debt limit under lower inflation is thereby most beneficial for indebted agents. Comparing the effects of initial debt deflation with the borrowing constraint effects, indicates that the total welfare effect is positive for less indebted agents with $s \in [-0.1, 0)$. These agents do not face a binding borrowing constraint. Yet, they assign a positive probability of being constrained in the future such that a relaxation of the effective debt limit is beneficial for them. For these agents, the borrowing constraint effects outweighs the effects of initial debt deflation. For highly indebted agents (s_I) the latter effect dominates the former, while lenders unambiguously gain from the inflation reduction. The right hand panel in the last row of Figure 5 shows the welfare effects within the four wealth sets, ΔW_{s_x}. Computing the contribution to the total welfare effects over the entire population, shows that the aggregate welfare falls due to the effects of initial debt deflation by $\Delta W(ID) = -0.283\%$ and increases due to the borrowing constraint effects by $\Delta W(BC) = 0.234\%$. Hence, the decline of aggregate welfare due to effects of initial debt deflation is reduced by 83% via the novel borrowing constraint effect, such that the total aggregate welfare effect is just slightly negative, $\Delta W = -0.049\%$.

Sensitivity analysis To assess the sensitivity of these results, we compute corresponding results for a shorter maturity, for an increase instead for a reduction in the inflation rate by 4%, and for a lower autocorrelation of idiosyncratic income. Notably, the wealth distribution is not unaffected by these experiments, from which we abstract in the following discussion, for convenience. Reducing the debt maturity $1/\theta$ from 2 to 1 periods essentially reduces all effects in a proportional way (see left hand column of Figure 6), keeping their relative magnitudes unchanged. As in related studies (see Doepke and Schneider, 2006), effects of initial debt deflation induced by non-transitory inflation changes are

[24] Specifically, we compute the per capital welfare effects for each wealth set, i.e. $[\sum_{s_x,y} \lambda_2(s,y) g(s,y)] / \sum_{s_x,y} \lambda_2(s,y)$ for $s_x \in \{s_I, s_{II}, s_{III}, s_{IV}\}$ and proceed as described above to separate the effects of initial debt deflation from the borrowing constraint effects.

Figure 6: Welfare aggregated for four wealth sets with different maturities (left column) and different inflation rates (right column)



more persistent and amplified under longer-term nominal debt.[25] At the same time, the borrowing constraint effects also increase with higher maturities, as they increase the likelihood of borrowers to be unconstrained at maturity (similar to a lower autocorrelation of income). Increasing the inflation rate by 4% to 6% leads to welfare effects that are qualitatively symmetric to the effects of the inflation reduction to -2% (see right hand column of Figure 6). Yet, the size of all effects under higher inflation are smaller ($\Delta W_{6\%} = 0.04\%$) compared to the effects under an equally-sized inflation reduction. On the one hand, a higher inflation rate reduces the effective debt limit. On the other hand, an increase in inflation reduces the value of beginning-of-period debt $-s/\pi$. In total, the distortion induced by the borrowing constraint decreases with the inflation rate, such that the welfare effects of initial debt deflation as well as of the borrowing constraint are smaller under higher inflation rates.[26]

Notably, the specification and parametrization of the idiosyncratic income process is not undisputed. Guvenen (2007) for example suggests an income process which leads to much lower estimates for the autocorrelation of idiosyncratic income. To assess the impact of these estimates, we adjust the income process including the income states and we recalibrate relevant parameters. We therefore apply Guvenen's (2007) estimates $\rho = 0.821$ and $\sigma_\epsilon = 0.029$, and we set $\beta = 0.8968$ and $\gamma = 0.51$ to match the previously described targets. For this alternatively calibrated model specification, Figure 7 shows individual and aggregate welfare effects, which are comparable to the benchmark specification. Here, the separate welfare effects due to initial debt deflation and the borrowing constraint are $\Delta W(ID) = -0.108\%$ and $\Delta W(BC) = 0.126\%$, respectively. Apparently, the reduction in the inflation rate now leads to a (small) positive aggregate welfare effect, $\Delta W = 0.018\%$, consistent with the results summarized in Proposition 2.

Aggregate debt and real returns Finally, we examine the time path of aggregate beginning-of-period debt $-s/\pi$ and the real rate of return r^* in response to the inflation

[25] Similarly, monetary policy exert more persistent effects when nominal payments are fixed for longer terms as under mortgage contracts (see Gariga et a., 2017).

[26] This is indicated by the average value of the multiplier on the borrowing constraint ζ within the lowest wealth state s_I, which monotonically decreases from an inflation rate of -2%, to 2% and 6%. The values for $\overline{\zeta}_\pi = \Sigma_{s_I,y}\lambda_\pi(s,y)\zeta_\pi(s,y)/\left(\Sigma_{s_I,y}\lambda_\pi(s,y)\right)]$ are $\overline{\zeta}_{-2} = 1.048$, $\overline{\zeta}_2 = 1.014$, and $\overline{\zeta}_6 = 0.984$.

Figure 7: Individual welfare effects for three income states and welfare aggregated for four wealth sets under a calibration with lower autocorrelation of income

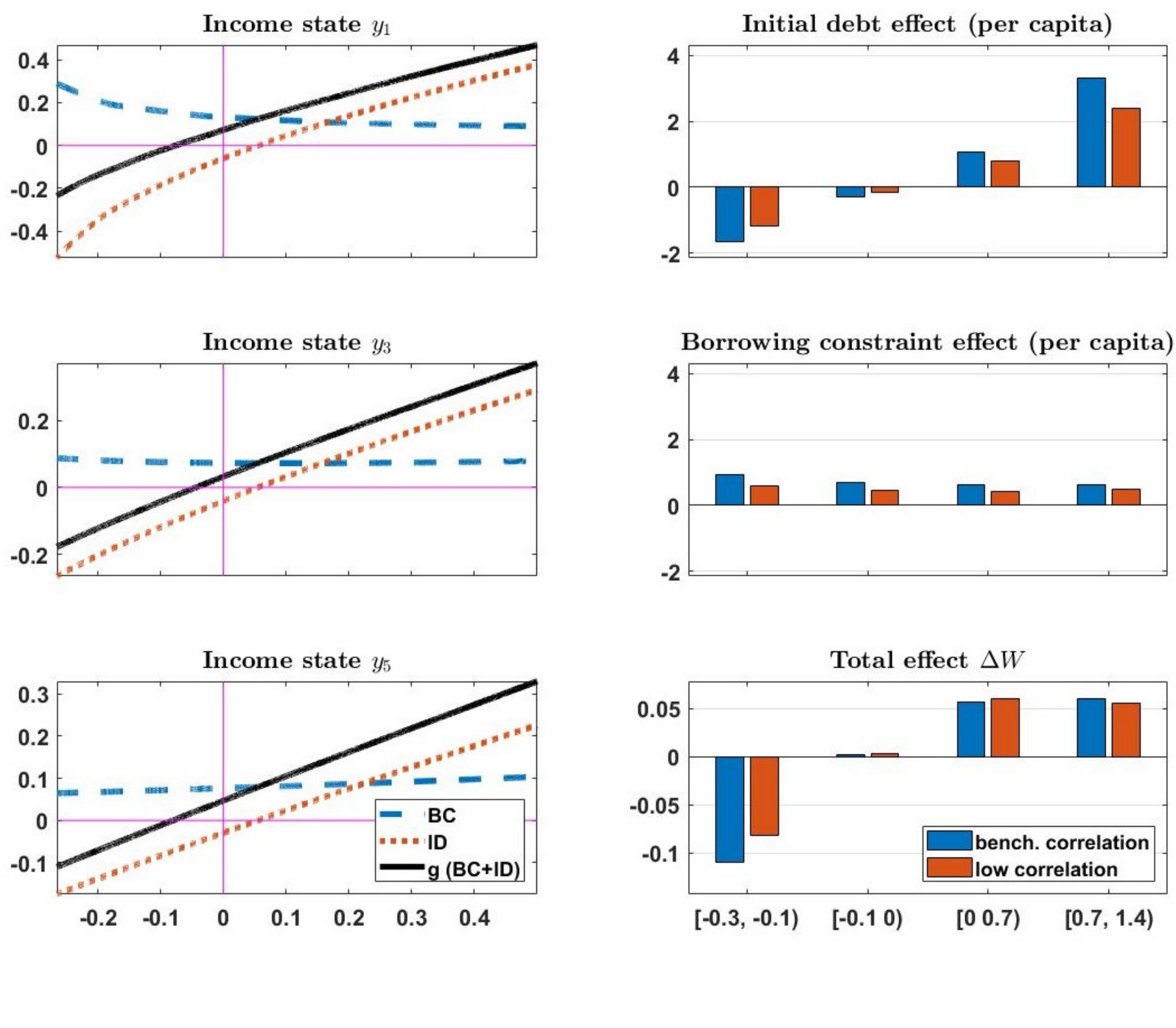
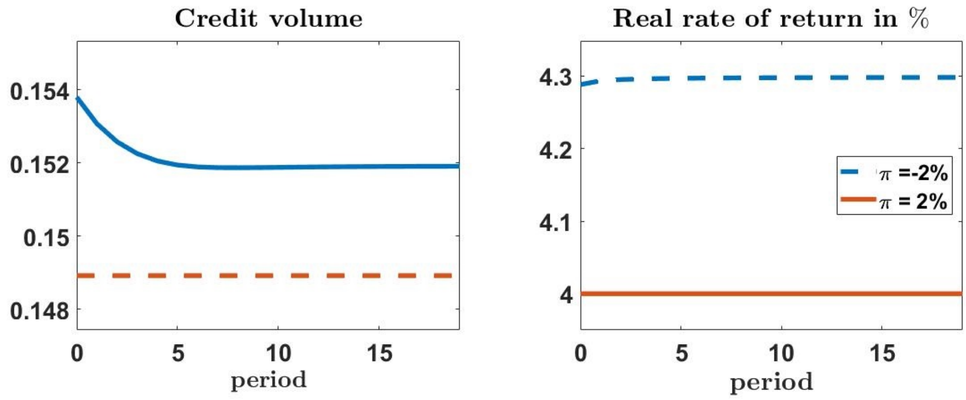


Figure 8: Paths of aggregate debt $-s/\pi$, and the real rate of return r^*



rate reduction (see Figure 8). When the inflation rate is reduced, the wealth distribution is initially consistent with an inflation rate of 2%. When the inflation rate is then reduced to -2%, the effective debt limit is raised, such that agents' access to external funds is less constrained and the aggregate credit volume increases on impact. From then onwards, the economy converges to a new stationary wealth distribution with a debt level that settles on an intermediate level. Given that aggregate debt $-s/\pi$ is higher under a lower inflation rate, market clearing requires a higher real rate of return r^*, which under our benchmark calibration increases from 4% and converges to 4.3% (see right panel of Figure 8). This uninternalized change in the real rate of return tends to reduce the overall welfare impact of the borrowing constraint effects.

4 Conclusion

We analyze how financial frictions contribute to redistributive effects of monetary policy. We explore a novel mechanism of monetary non-neutrality, which is based on borrowing constraints related to current income. Such limits for unsecured debt, for which broad empirical evidence exists, do not account for expected price changes until maturity, implying that monetary policy can alter the real terms of borrowing. A reduction in inflation tends to increase the maximum amount of debt that can be issued, while it also raises the beginning-of-period stock of debt to be repaid. The impact of inflation depends on the probability of borrowers to be unconstrained at maturity. The lower this probability is, the smaller is the beneficial effect of lower inflation for borrowers. The debt limit effect is opposed to debt deflation effects when borrowers are initially indebted. The overall effect is therefore ex-ante ambiguous and depends on the initial debt/wealth position as well as the willingness to borrow. We show that lower inflation particularly benefits agents with low initial debt by relaxing effective borrowing constraints, whereas highly indebted borrowers suffer from the dominant debt deflation effect. A reduction of the inflation rate can nonetheless enhance aggregate welfare, specifically, when the autocorrelation of idiosyncratic income is relatively low.

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Paper No. 22285.

A Proof of Proposition 2

We start by establishing the first claim of the proposition. Under a constant inflation rate, the equilibrium exhibits no time variation, such that we can neglect time indices. Substituting out the interest rate with (13), which can – by using (16) – be rewritten as $1/R = (\beta/\pi)\,[p_{21}\frac{\delta-2c_b}{\delta+2c_b-2(y_1+y_2)} + p_{22}]$, in the borrower's budget constraint (12), implying

$$c_b = (b/\pi)\left[\beta p_{21}\frac{\delta - 2c_b}{\delta + 2c_b - 2(y_1 + y_2)} + \beta p_{22} + p_{21} - p_{11}\right] + y_1, \tag{17}$$

where the fraction on the RHS is strictly decreasing in c_b. Thus, a lower inflation rate increases c_b if the term in the squared brackets in (17) is positive, i.e.

$$\beta\left\{p_{21}\frac{\delta - 2c_b}{\delta + 2c_b - 2(y_1 + y_2)} + p_{22}\right\} + 2p_{21} - 1 > 0, \tag{18}$$

where we used $p_{21} + p_{11} = 1$. The term in the curly brackets in (18) is larger than one under a binding borrowing constraint, since $p_{21} + p_{22} = 1$ and the marginal utility of the representative borrower is larger than the marginal utility of the representative lender implying $\frac{\delta-2c_1}{\delta+2c_1-2(y_1+y_2)} > 1$. Thus, $\beta + 2p_{21} - 1 > 0$ is sufficient to satisfy the inequality (18). In this case, a lower inflation rate increases c_b. Given that $(\delta - 2c_{b,t}) > (\delta - 2c_{l,t}) \Leftrightarrow c_b < c_l$, an increase in c_b and thus a decrease in c_l by the same amount causes a reduction of the gap between the marginal utility of the representative borrower and the marginal utility of the representative lender. Hence, aggregate welfare, measured as $(1-\beta)^{-1}[(\delta c_b - c_b^2) + (\delta c_l - c_l^2)]$, unambiguously increases if $p_{12} > (1-\beta)/2$.

To establish the claim regarding first best, we use that $c_{b,t} = c_{l,t}$ holds under first best. Then, (15) implies

$$-s = R(y_2 - y_1 + 2(b/\pi)(p_{11} - p_{21}))/2 \leq b, \tag{19}$$

where the inequality is due to the non-binding borrowing constraint under first best. Under first best, (13) further implies $R/\pi = 1/\beta$. Substituting out inflation with the latter in (19), gives $R \leq 2b\frac{1+(p_{21}-p_{11})/\beta}{y_2-y_1}$, which together with the ZLB imply $1 \leq 2b\frac{1+(p_{21}-p_{11})/\beta}{y_2-y_1}$ for monetary policy to be able to implement first best. If however $1 > 2b\frac{1+(p_{21}-p_{11})/\beta}{y_2-y_1}$ monetary policy cannot implement first best due to the ZLB. ■

B Appendix to the calibrated model

B.1 Data on household debt

The ratios of debt to income for different income quintiles are calculated as follows. For income we use average after tax income in the income quintiles in 2004 (in 2004 dollars) taken from CBO (www.cbo.gov/sites/default/files/109th-congress-2005-2006/reports/ EffectiveTaxRates2006.pdf) which we denote by Av. ATI. (see second column of Table 1).

Income Quintile	Av. ATI	Av. IL w.o. VIL	debt to income
Q1	14.7k	7.16k	0.49
Q2	32.7k	8.52k	0.26
Q3	48.4k	6.90k	0.14
Q4	67.7k	7.88k	0.12
Q5	155.2k	11.13k	0.07

Table 1: Average after tax income, average value of these debt holdings, and debt-to-income for income quintiles in 2004 (in 2004 dollars)

For debt we use the following component of installment loans taken from the SCF 2016 (where dollar variables are inflation-adjusted to 2004 dollars): All installment loans (which exclude loans secured by residential property) minus vehicle installment loans. For every income quintile, we then use the average value of these debt holdings of those households who hold this type of debt. We then denote this type of debt by Av. IL w.o. VIL (see third column of Table 1) and it is calculated by "Av. IL w.o. VIL" = "Av. IL" - "Av. VIL" * "% w. VIL" / "% w. IL", where Av. IL denotes the average value of all installment holdings of households who hold this type of debt in a given income quintile, Av. VIL is the average value only of vehicle installment loans, % w. IL denotes the fraction of households who have an installment loan in a given income quintile, and % w. VIL is the fraction of households who hold only vehicle installment loans. The debt to income ratios we use (see Figure 2) are then given by the ratio of average after tax income and installment loans net of vehicle loans (see fourth column of Table 1).

B.2 Transition matrix

The transition matrix of idiosyncratic income with the conditional probabilities $\mathcal{P}(a_l|a_k)$ is given by

$$\mathcal{P} = \begin{pmatrix} 0.767 & 0.207 & 0.025 & 0.001 & 10^{-6} \\ 0.207 & 0.496 & 0.253 & 0.043 & 0.001 \\ 0.025 & 0.253 & 0.446 & 0.253 & 0.0245 \\ 0.001 & 0.043 & 0.253 & 0.496 & 0.207 \\ 10^{-6} & 0.001 & 0.025 & 0.207 & 0.767 \end{pmatrix}$$

B.3 Calculation of the stationary equilibrium under a given inflation rate

Under a given inflation rate π, we calculate the decision rules and the time-invariant distribution at a given issuance price of a unit bond $Q = Q'$ by using an endogenous grid point method (see Carroll, 2006) combined with time iteration and we calculate the stationary equilibrium issuance price by a bisection method as follows:.

I. For the bisection method we need (i) a value for the issuance price denoted by Q^l, i.e. $Q = Q' = Q^l$, at which $\sum_{s,y} \lambda(s,y)s'(s,y) > 0$ and (ii) a value for the issuance price denoted by Q^h, i.e. $Q = Q' = Q^h$, at which $\sum_{s,y} \lambda(s,y)s'(s,y) < 0$. The stationary equilibrium issuance price satisfying $\sum_{s,y} \lambda(s,y)s'(s,y) = 0$ is then in the interval $\left(Q^l, Q^h\right)$. To find a value that satisfies the condition in (i) we choose a relatively low value for the issuance price, calculate steps III-IV and check the condition $\sum_{s,y} \lambda(s,y)s'(s,y) > 0$. If this condition is not satisfied, we repeat steps III-IV with lower issuance prices until we have found a value that satisfies the condition in (i). Proceed analogously for Q^h.

II. Calculate a guess for the stationary equilibrium issuance price Q^0 by $Q^0 = 0.5\left(Q^l + Q^h\right)$.

III. Calculate for Q^0 the consumption policy function $c(s,y)$ and the wealth policy function $s'(s,y)$ with an endogenous grid point method combined with time iteration neglecting market clearing for loans (see below).

IV. Given the wealth policy function $s'(s,y)$, compute the implied stationary distribution $\lambda(s,y)$ (see below).

V. Check market clearing for loans. Choose a pararmeter $\epsilon > 0$, which is relatively small. If $|\sum_{s,y} \lambda(s,y)s'(s,y)| < \epsilon$, stop: $Q = Q^0$ is the equilibrium issuance price. If $\sum_{s,y} \lambda(s,y)s'(s,y) > \epsilon$, set $Q^l = Q^0$ and go back to step II. If $\sum_{s,y} \lambda(s,y)s'(s,y) < \epsilon$, set $Q^h = Q^0$ and go back to step II.

The endogenous grid point method combined with time iteration for a given issuance price $Q = Q' = Q^0$ is computed as follows:

1. Discretize next period wealth space $s' = \left\{s'_1, s'_2, ..., s'_{y4}, ..., s'_{y3}, ..., s'_{y2}, ..., s'_{y1}, ..., s'_m\right\}$, $s'_i < s'_{i+1}$ with $s'_1 = s'_{y_5} = -\gamma y_5$ and $s'_{y_i} = -\gamma y_i$. Thus, the discretized 2-dimensional state space is given by $\{s'_1, s'_2, ..., s'_m\} \times \{y'_1, y'_2, ..., y'_n\}$, where y'_k, $k = 1, ..., n$, are the possible income states. Choose a stopping rule parameter $\epsilon^{egm} > 0$. Note that the calculation of a stationary equilibrium in I-V requires a bounded wealth space where the maximum value denoted by s_{max} satisfies $s'(s_{max}, y) \leq s_{max}$ for all y under the wealth policy function $s'(s,y)$ calculated by the endogenous grid point method for a given issuance price Q^0. The highest value s_m in our wealth space is a guess for a state that satisfies this condition. We check this condition after having calculated the policy functions at a given issuance price Q^0 (see 5).

2. Make a guess for next period's consumption policy function $(c')^0(s'_i, y'_k)$, where $k \in \{1, ..n\}$ and the guess is computed by $(c')^0(s'_i, y'_k) = -Q^0(s'_i - (1-\theta)s'_i/\pi) + \theta s'_i/\pi + y'_k$, at all states in the discretized state space.

3. Calculate a guess for current period's consumption policy function $c^0(s_i, y_k)$ (using two auxiliary functions $\hat{c}(s'_i, y_k)$ and $\hat{s}(s'_i, y_k)$):

   - Use $(c')^0(s'_i, y'_k)$ to compute a guess for current period consumption using $\hat{c}(s'_i, y_k)$ for future period wealth s'_i and some current period income y_k by using the Euler equation:

$$\hat{c}(s'_i, y_k) =$$
$$\left(\frac{1-\theta+\theta/Q^0}{\pi/\beta} \left(p_{k1} \left(c'\right)^0 (s'_i, y'_1)^{-\sigma} + p_{k2} \left(c'\right)^0 (s'_i, y'_2)^{-\sigma} + ... + p_{kn} \left(c'\right)^0 (s'_i, y'_n)^{-\sigma} \right) \right)^{-1/\sigma}$$

where $s'_i \geq s'_{y_k}$ at today's income state y_k due to the borrowing constraint.

- Use the budget constraint and the auxiliary function $\hat{c}(s'_i, y_k)$ to compute current period wealth $\hat{s}$ for s'_i and y_k:

$$\hat{s}(s'_i, y_k) = \left(\hat{c}(s'_i, y_k) + Q^0 s'_i - y_k\right) \pi / \left(Q^0(1-\theta) + \theta\right)$$

- Calculate current period's consumption policy function at $(s_i, y_k) \in \{s_1, s_2, ..., s_m\} \times \{y_1, y_2, ..., y_n\}$ where the grid for today's wealth states is the next period's grid, i.e. $s_i = s'_i$, as follows:
  - The beginning-of-period wealth $\hat{s}(s', y_k)$ for $s' = -\gamma y_k$ is the highest wealth position in the discretized wealth space at which a household with income y_k borrows the maximum amount.
  - At $s_i \leq \hat{s}(-\gamma y_k, y_k)$, a household with the same income y_k but with beginning-of-period wealth s_i that is smaller or equal to $\hat{s}(-\gamma y_k, y_k)$ is borrowing constrained as well. The current period's consumption policy function at (s_i, y_k) is then computed by

$$c^0(s_i, y_k) = Q^0 \left(\gamma y_k + (1-\theta) s_i/\pi\right) + \theta s_i/\pi + y_k$$

    and end-of-period wealth is given by

$$\left(s'\right)^0 (s_i, y_k) = -\gamma y_k.$$

  - At $s_i > \hat{s}(-\gamma y_k, y_k)$, the borrowing constraint is not binding at beginning-of-period wealth s_i and income y_k in the current period. The current period's consumption policy function c^0 at (s_i, y_k) is then calculated using the implicit definition $\widetilde{c}^0(\hat{s}(s'_i, y_k), y_k) = \hat{c}^0(s'_i, y_k)$ where s'_i is today's choice for future beginning-of-period wealth when today's income is y_k while

$\hat{s}(s'_i, y_k)$ is today's beginning-of-period wealth which under current income y_k leads to this choice of s'_i. Then, $c^0(s_i, y_k)$ is computed by a linear interpolation of $\widetilde{c}^0(\hat{s}, y)$ at (s, y), where s takes on-grid values. The wealth policy function at (s_i, y_k) is then computed by using the budget constraint

$$(s')^0(s_i, y_k) = -\left(c^0(s_i, y_k) - \left(Q^0(1-\theta) + \theta\right) s_i/\pi - y_k\right)/Q^0.$$

- IF $||(c')^0(s' = s, y' = y) - c^0(s, y)|| < \epsilon^{egm}(1 + ||c^0(s, y)||)$, stop. Under the current guess for the issuance price Q^0, the policy function for consumption is then given by $c(s, y) = c^0(s, y)$ and the policy function for wealth is given by $s'(s, y) = (s')^0(s, y)$
  ELSE $(c')^0 = c^0$ and start again step 3.

4. IF $s'(s_m, y) \leq s_m$ for all y, stop.
   ELSE choose a higher value s_m and go back to step 1.

The stationary distribution for given policy functions is computed by calculating the normalized eigenvalue of the Markov transition matrix:

1. We add further wealth states to get a finer grid than the one used for the calculation of the policy functions (from 5 to 100 thousand grid points for s) and we calculate the wealth policy function values for the new states.

2. Calculate the transition probability of being in the state (s_j, y_l) in the next period if the current state is (s_i, y_k) and denote it by $P((s_i, y_k), (s_j, y_l))$. This probability is computed by $P((s_i, y_k), (s_j, y_l)) = \mathcal{P}(y_l|y_k) * I(s'(s_i, y_k) = s_j)$, where $I(s'(s_i, y_k) = s_j) = 1$ if $s'(s_i, y_k) = s_j$ and 0 otherwise. The Markov transition matrix is then given by the transition probabilities $P((s_i, y_k), (s_j, y_l))$ for all combinations of states.

3. Compute the eigenvector of the transition matrix associated with the largest eigenvalue (which is one). The stationary distribution on the grid is then given by the normalization of this eigenvector.

B.4 Calculation of the transition path to the new stationary equilibrium

At the beginning of period 0 the economy is in the stationary equilibrium under an inflation rate of 2% with the beginning-of-period distribution of wealth s induced by this inflation rate. In period 0 then the inflation rate unexpectedly and permanently changes to -2%. The economy then leaves the old stationary equilibrium in period 0 and converges to the new stationary equilibrium under an inflation rate of -2%. The transition path is computed as follows (see e.g. Rios-Rull, 1999):

- Calculate the stationary equilibria for the two inflation rates of 2% and -2% as described above and denote the respective stationary distributions by $\lambda_{2\%}$ and $\lambda_{-2\%}$.
- The beginning-of-period distribution in period t of the transition path is denoted λ_t. In period 0, this distribution is given by $\lambda_0 = \lambda_{2\%}$. The beginning-of-period distribution after the economy has converged into the new stationary equilibrium is denoted λ_∞ and given by $\lambda_\infty = \lambda_{-2\%}$.
- Calculate the transition path:

  1. Assume that the transition into the new stationary equilibrium takes T periods. This implies $\lambda_T = \lambda_\infty$.
  2. Find two price paths $Q^{l,1} = \{Q_t^{l,1}\}_{t=0}^T$ and $Q^{h,1} = \{Q_t^{h,1}\}_{t=0}^T$ with $Q_t^{l,1} < Q_t^{h,1}$ for all $t \leq T$ that satisfy (i) $|\sum_{s,y} \lambda_t^Q(s,y) s_{t+1}^Q(s,y)| > 0$ at $Q_t^{l,1}$ and (ii) $|\sum_{s,y} \lambda_t^Q(s,y) s_{t+1}^Q(s,y)| < 0$ at $Q_t^{h,1}$ for all $t \leq T$ where λ_t^Q and $s_{t+1}^Q(s,y)$ denote the distribution and wealth policy function in period t of the transition path under a given price path. To find a price path that satisfies (i), we choose a path with relatively low values for the issuance prices, calculate steps 4-5 and check the condition (i) for all $t \leq T$. If this condition is not satisfied, we repeat steps 4-5 with a lower price path until we have found a path that satisfies the condition (i). Proceed analogously for Q_t^h. Choose stopping rule parameters $\epsilon^s > 0$ and $\epsilon^\lambda > 0$.

3. Denote the current iteration step by i. Calculate a price path $\{\tilde{Q}_t\}_{t=0}^T$ with $\tilde{Q}_T$ given by the value of the stationary equilibrium induced by the inflation rate of -2% and $\tilde{Q}_t = 0.5(Q_t^{l,1} + Q_t^{h,1})$ for $t < T$. In iteration step i, a guess for the equilibrium sequence of issuance prices of the transition path $\{\hat{Q}_t^i\}_{t=0}^T$ is then calculated by:
   - If $i = 1$, $\{\hat{Q}_t^i\}_{t=0}^T = \{\tilde{Q}_t\}_{t=0}^T$
   - If $i > 1$, $\{\hat{Q}_t^i\}_{t=0}^T = \varphi\{\hat{Q}_t^{i-1}\}_{t=0}^T + (1-\varphi)\{\tilde{Q}_t\}_{t=0}^T$ with $\varphi \in [0,1)$
4. Since we know $c_T(s,y)$, which is given by the policy function of the new stationary equilibrium, and have a guess $\{\hat{Q}_t^i\}_{t=0}^T$, we can solve backwards for the policy functions in period $t = 0, ..., T-1$ of the transition path at the given price path $\{\hat{Q}_t^i\}_{t=0}^T$. We denote these policy functions by $\{c_t^Q(s,y), s_{t+1}^Q(s,y)\}_{t=0}^T$ where $c_T^Q(s,y)$ and $s_{T+1}^Q(s,y)$ are the policy functions of the new stationary equilibrium.
5. Use the policy functions $\{s_{t+1}^Q(s,y)\}_{t=0}^{T-1}$ and λ_0 to iterate the distribution forward to get a path for the distribution at the given price path $\{\hat{Q}_t^i\}_{t=0}^T$. We denote this path for the distribution by $\{\lambda_t^Q\}_{t=0}^T$ with $\lambda_0^Q = \lambda_0$.
6. Use $\{\lambda_t^Q\}_{t=0}^T$ to compute $\hat{A}_t = \sum_{s,y} \lambda_t^Q(s,y)s_{t+1}^Q(s,y)$ for $t = 0, ..., T$. Check for debt market clearance: If

$$\max_{0 \le t \le T} \left|\hat{A}_t\right| < \epsilon^s$$

   go on. If not, set $Q_t^{l,i+1} = \varpi Q_t^{l,i} + (1-\varpi)\hat{Q}_t^i$ with $\varpi \in [0,1)$ in periods in which $\hat{A}_t > \epsilon^s$ and $Q_t^{h,i+1} = \varpi Q_t^{h,i} + (1-\varpi)\hat{Q}_t^i$ in periods in which $\hat{A}_t < \epsilon^s$ and go back to step 3.
7. Check for $\left\|\lambda_T^Q - \lambda_T\right\| < \epsilon^\lambda$. If yes, the transition converges smoothly into the new stationary equilibrium, $\{Q_t\}_{t=0}^T = \{\hat{Q}_t^i\}_{t=0}^T$ is the equilibrium price path and the equilibrium policy functions are given by $\{c_t, s_{t+1}\}_{t=0}^T = \{c_t^Q, s_{t+1}^Q\}_{t=0}^T$. If not, go back to step 1 and start again with a higher T.
8. After having calculated the transition path for the policy functions and wealth distribution, we calculate the transition path for the value functions $\{v_t(s,y)\}_{t=0}^T$.

Denote the value function in the stationary equilibrium induced by an inflation rate $\pi = -2\%$ ($\pi = 2\%$) by $\overline{v}_{-2}$ ($v_{2\%}$). The value function in period T is then given by $v_T = \overline{v}_{-2}$. We solve for the value functions in periods $t = 0, ..., T-1$ backwards from period T on by

$$v_t(s_i, y_k) = u(c_t(s_i, y_k)) + \beta \sum_{l=1}^{5} p_{k,l} v_{t+1} \left(s_{t+1}(s_i, y_k), y'_l \right)$$

using v_T and policy functions c_t and s_{t+1} where y'_l for $l = 1, ..., 5$ denotes the possible income states in the next period $t+1$.

- Note that $v_{-2}(s, y)$ is the expected lifetime utility in period 0 of a household with income y and beginning of period 0 wealth s who has just been hit by the change in the inflation rate to -2%. This lifetime utility takes into account all the transition dynamics which the household is going to live through while $v_2(s, y)$ gives the expected lifetime utility in period 0 of a household with the same income y and beginning of period 0 wealth s but who lives in an economy under an unchanged inflation rate of $\pi = 2\%$. If $v_{-2}(s, y) > (<) v_2(s, y)$, a household in state (s, y) in period 0 benefits (looses) under the reduction in the inflation rate.

C Additional figures

Figure 9: Welfare (relative to a reference economy with $\pi = 1.02$ and $\gamma = 0.487$, in percent) and consumption share of relatively patient agents

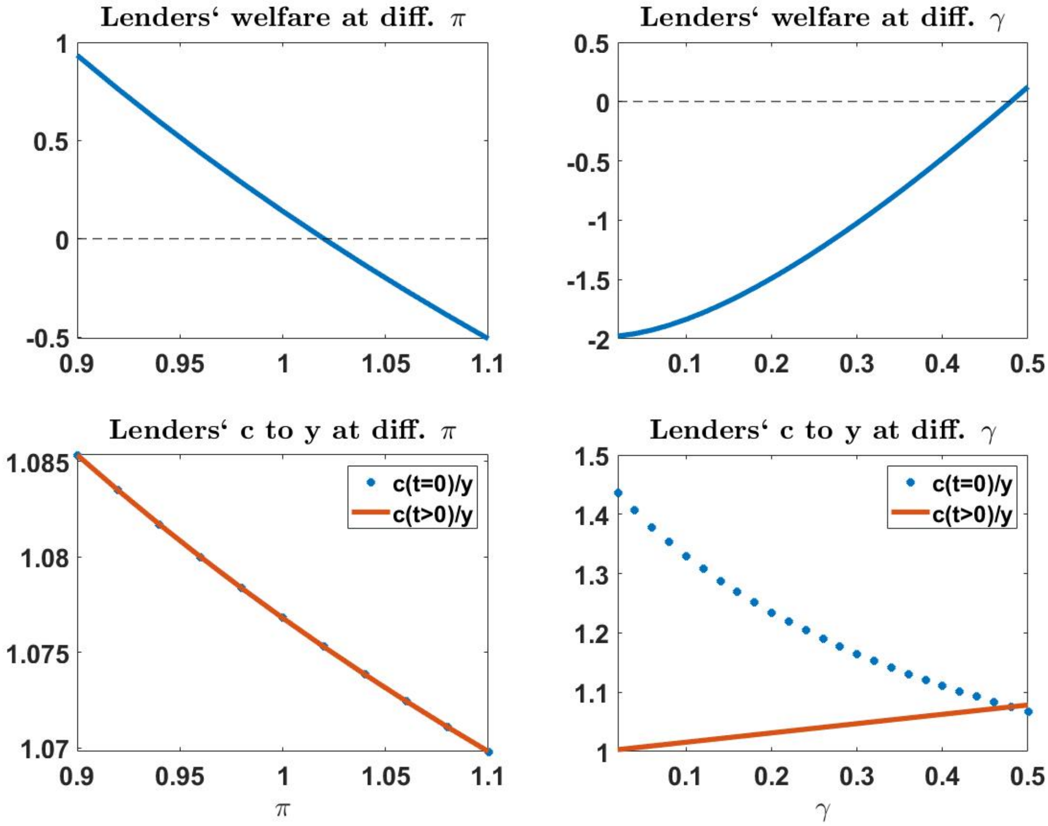

Figure 10: Policy functions for consumption and savings in period 0

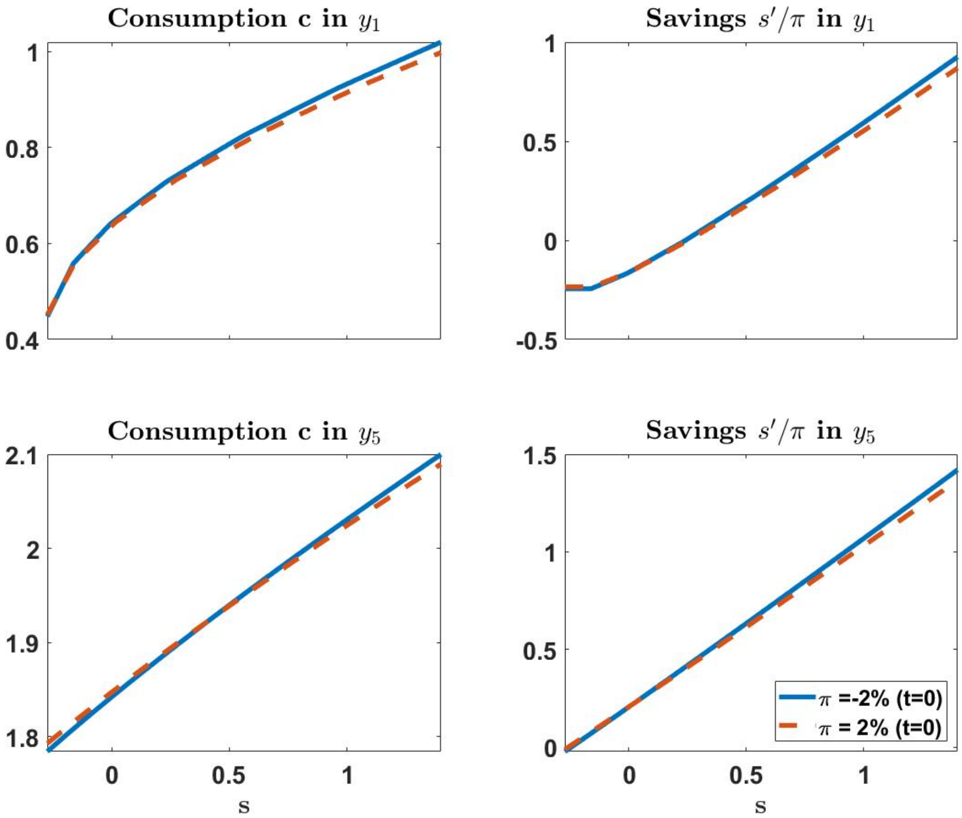

Figure 11: Policy functions for consumption and savings for stationary wealth distributions