Optimal income taxation with labor supply responses at two margins: When is an Earned Income Tax Credit optimal?

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Abstract

This paper studies optimal non-linear income taxation in an empirically plausible model with labor supply responses at the intensive (hours, effort) and the extensive (participation) margin. In this model, redistributive taxation gives rise to a previously neglected trade-off between two aspects of efficiency: To reduce the deadweight loss from distortions at the extensive margin, the social planner has to increase distortions at the intensive margin and vice versa. Due to this trade-off, minimizing the overall deadweight loss requires to distort labor supply by low-skill workers upwards at both margins. Building on these insights, the paper provides conditions under which social welfare is maximized by an *Earned Income Tax Credit* with negative marginal taxes and negative participation taxes at low income levels. Numerical simulations suggest that the optimal tax credit can be quantitatively comparable to or even larger than the current *EITC* in the US.

Keywords: Optimal income taxation, Extensive margin, Intensive margin, Earned Income Tax Credit

JEL classification: H21; H23; D82

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1 Introduction

Governments in all developed countries use labor income taxes and income-based transfers to redistribute resources from the rich to the poor. The properties of these tax-transfer schemes differ substantially across countries, though, especially with respect to the treatment of low incomes. In most European countries, public transfers are monotonically decreasing in the market incomes of the recipients. In the US and some other countries, in contrast, transfers to the working poor are higher than those to the unemployed and (sometimes) increasing in earned income. In particular, the *Earned Income Tax Credit (EITC)* in the US entails both negative marginal taxes and negative participation taxes for low-income earners.[1] On the one hand, there seems to be a growing consensus among political practitioners that the *EITC* is an effective instrument for fighting poverty and should be further expanded.[2] On the other hand, economists have so far struggled to rationalize the use of tax-transfer schemes with these properties.[3] The present paper fills this gap by providing sufficient conditions as well as a novel explanation for the optimality of an *EITC* with negative marginal taxes and negative participation taxes.

The common approach to determine the optimal income tax involves, first, the definition of a welfare function that provides a rationale for redistribution from the rich to the poor, and second, the maximization of this welfare function over the set of (non-linear) income taxes that satisfy the government's budget constraint, taking into account the agents' labor supply responses. A large set of papers show that an *EITC* cannot be optimal if labor supply responds at the intensive (hours, effort) margin only. A smaller set of papers find that negative participation taxes may be optimal if labor supply responds at the extensive (participation) margin only.[4] Both classes of models are inconsistent with the empirical evidence that labor supply responds at the intensive margin *as well as* the extensive margin, however: "the world is obviously a mix of the two models" (Saez 2002: p. 1054). More specifically, empirical studies consistently find that extensive-margin responses are particularly important at the bottom of the income distribution: The participation elasticity of

[1]The participation tax function is commonly defined as the difference between the net taxes to be paid at income levels y and 0 (for any positive income y). Nichols & Rothstein (2015) provide a detailed description of the details of the *EITC* and a comprehensive review of the recent literature studying its labor supply effects.

[2]Most prominently, both President Obama and Paul Ryan – then Republican Chairman of the House of Representatives Budget Committee – proposed to roughly double the maximum *EITC* payments for childless workers (see Executive Office 2014, House Budget Committee 2014).

[3]In particular, most previous papers find that negative marginal income taxes cannot be optimal. I comment on the most important exceptions in Section 2 below.

[4]In these models, marginal taxes have no effect on the agents' behavior. Therefore, the papers typically do not study their optimal signs.

low-income earners is both larger than their elasticity of hours worked, and larger than the participation elasticity of medium-income and high-income earners (see, e.g., Juhn et al. 1991, 2002, Meghir & Phillips 2010).[5]

The present paper investigates optimal income taxation in a framework that is consistent with these empirical patterns. In particular, I study a two-dimensional screening model in which the agents face both marginal costs of providing output as in Mirrlees (1971) and fixed costs of working as in Diamond (1980). The agents are privately informed about their fixed costs of working and their skills, where the latter determine the marginal costs of output provision. To make the model tractable, I assume additive separability between the fixed-cost component and the other components of the utility function, thereby following the random-participation approach by Rochet & Stole (2002).[6] The analysis focuses on the empirically and economically relevant cases in which, first, society has standard concerns for redistribution from higher-income earners to lower-income earners and, second, participation elasticities are decreasing over the skill dimension.

The paper contributes in four ways to the literature on optimal income taxation. First, it derives two novel theoretical results on the optimality of an *EITC*. The first result identifies two properties that the optimal income tax satisfies whenever society has well-behaved redistributive concerns: Optimal participation taxes are strictly negative up to some income threshold $y_k \geq 0$ and positive above that threshold, and optimal marginal taxes are strictly negative at some income levels below y_k and positive everywhere else.[7] The paper is hence the first to show that negative marginal taxes can only be optimal (i) at the bottom of the income distribution and (ii) in combination with a negative participation tax, i.e., as part of an *EITC*. The second result provides sufficient conditions for the optimality of such an *EITC*, expressed in terms of the model's primitives: utility functions, type distributions, and the properties of the social welfare function. In particular, optimal marginal taxes and participation taxes are negative at low income levels if society has strong concerns for redistribution from the rich to the poor, but only limited concerns for local redistribution from the poor to the very poor. Crucially, this result does not depend on whether labor supply responds more strongly on the intensive margin or on the extensive margin; it holds whenever labor supply responds at both margins.[8]

[5] For additional empirical evidence on how participation elasticities vary across the population, see Meyer & Rosenbaum (2001), Eissa & Hoynes (2004), Immervoll et al. (2007), Blau & Kahn (2007) and the surveys by Hotz & Scholz (2003), Eissa & Hoynes (2006), McClelland & Mok (2012).

[6] The same assumption is also used in the closely related paper by Jacquet et al. (2013).

[7] The optimal income tax involves an *EITC* if and only if the threshold y_k is strictly positive.

[8] More precisely, these results hold if labor supply elasticities at both margins differ from zero, and low-income earners respond more strongly at the extensive margin than high-income earners.

Surprisingly, only a few previous papers have studied optimal income taxation in a model with labor supply responses at both margins. Most prominently, Saez (2002) demonstrates the differences between both one-margin models and provides preliminary results for a two-margin model. In particular, he shows that the optimal marginal taxes for low-income workers can be negative if the optimal participation taxes (and the participation elasticities) of higher-income earners are sufficiently large (Saez 2002, p. 1055). He does not clarify in which cases this condition is met.[9] Nevertheless, the general perception of Saez' results seems to be that an *EITC* can be optimal if and only if labor supply responds more strongly at the extensive margin than at the intensive margin (see, e.g., Brewer et al. 2010, Piketty & Saez 2013).[10] More recently, Jacquet et al. (2013) provide conditions that ensure the optimality of positive marginal taxes at all income levels, expressed in terms of the (endogenous) participation elasticities and social welfare weights. Both papers do not provide sufficient conditions for the optimality of negative marginal taxes (neither on the primitives of the model nor on labor supply elasticities and welfare weights).

Second, the paper conducts numerical simulations of the optimal tax schedule, which allow to assess the quantitative relevance of its theoretical results. For this purpose, I calibrate the model to the US economy, targeting empirical estimates from the previous literature and recent Current Population Survey (CPS) data. According to the simulation results, optimal marginal taxes for childless singles may be negative for all incomes below $\$15,000$, where a maximal tax credit of $\$1,700$ is reached. Optimal participation taxes may even stay negative up to incomes around $\$32,000$. These numerical results suggest that the optimal tax credit may be considerably larger than the current *EITC* for childless singles in the US.[11] It is worth emphasizing that the simulations only consider social objectives that give rise to concerns for redistribution from the rich to the poor, i.e., for which marginal welfare weights are monotonically decreasing over the income distribution.[12]

Third, the paper provides a novel intuition for the potential optimality of an *EITC*. In particular, it shows that the optimality of negative marginal taxes is

[9] In the model by Saez (2002), optimal marginal taxes and optimal participation taxes cannot be determined separately, as they jointly depend on the welfare weights and the intensive and extensive elasticities. The same is true in model studied below.

[10] The present paper contradicts this perception, showing that an *EITC* can be optimal whenever labor supply responds at both margins.

[11] For childless singles, *EITC* payments increase up to an earned income around \$6.600, where the peak of \$500 is reached (just offsetting payroll taxes in this income range). *EITC* payments are larger for single and married parents, for which the model is not calibrated.

[12] In contrast, the simulations in Saez (2002) and Jacquet et al. (2013) find optimal marginal taxes to be positive everywhere (while optimal participation taxes are sometimes negative at low incomes). While they focus on cases with equally strong redistributive concerns over the entire income range, I consider cases with weak concerns for redistribution among the poor.

driven by an inherent trade-off between labor supply distortions at both margins, which has not been elucidated in the previous literature.[13] The following thought experiment helps to understand this trade-off and its implications. Consider an economy populated by agents who differ both in their skills (very low, low, or high) and in their fixed costs of working, so that some agents in each skill group choose to remain unemployed for each tax schedule. Assume that the social planner wants to redistribute some fixed, strictly positive amount of resources from the rich (high-skilled workers) to the poor (unemployed agents, very-low-skill workers and low-skill workers) in such a way that efficiency is maximized, i.e., the deadweight loss from labor supply distortions at both margins is minimized. Hence, he does not care for how the resources are distributed among the poor. The properties of the efficiency-maximizing redistribution scheme can be explained in two steps.

For the first step, assume that the social planner only seeks to minimize the labor supply distortions at the extensive margin (given some amount of redistribution). If he increases the transfer to the unemployed, some workers in all three skill groups find it attractive to leave the labor market and save the fixed costs of working. If he increases the transfers to both groups of lower-skill workers, some unemployed agents in these skill groups find it attractive to enter the labor market, but none of the high-skill agents has an incentive to leave the labor market. Hence, the second option induces less distortions at the extensive margin. Accordingly, the efficiency-maximizing tax schedule involves higher transfers to both groups of lower-skill workers than to the unemployed, i.e., negative participation taxes.[14]

But how should these transfers be divided between both groups of lower-skill workers? To minimize the distortions at the extensive margin, the planner has to apply a version of the classical *inverse elasticity rule*. If the very-low-skill agents respond more elastically at the extensive margin than all higher-skilled agents (in line with the empirical evidence), the planner should pay smaller transfers to the very-low-skill workers than to the low-skill workers. For this purpose, he has to introduce negative marginal taxes in the relevant income range.

For the second step, assume that the social planner also seeks to minimize the labor supply distortions at the intensive margin. The lower-skilled workers respond to negative marginal taxes by increasing their output provision, so that labor supply becomes upwards distorted at the intensive margin. Hence, the social planner faces a trade-off between labor supply distortions at both margins: To reduce the

[13] While this trade-off is also present in the models by Saez (2002) and Jacquet et al. (2013), the authors do neither discuss the trade-off itself nor its relevance for the optimality of an *EITC*.

[14] These arguments are closely related to those in papers on optimal income taxation with labor supply responses at the extensive margin only (see, e.g., Saez 2002 and Christiansen 2015).

deadweight loss from upward distortions at the extensive margin, he has to increase the upward distortions at the intensive margin and vice versa. The optimal compromise between both types of distortions can only be decentralized by an *EITC* with negative participation taxes and negative marginal taxes.[15]

Fourth, the paper proposes a new strategy to analytically solve multi-dimensional screening models. The major problem in solving these models is that the set of binding incentive-compatibility (IC) constraints is *a priori* unclear. Jacquet et al. (2013) show that this problem can sometimes be circumvented. In particular, they identify conditions under which all downwards IC constraints along the skill dimension are binding and optimal marginal taxes are positive everywhere, just as in Mirrlees (1971). An *EITC* with negative marginal taxes can only be optimal in cases where at least some upwards IC constraints are binding, however. Hence, I have to develop a new approach to identify sufficient conditions for the optimality of an *EITC*.

The methodological innovation of this paper is to study a hybrid model with a continuous set of fixed cost types and a large but discrete set of skill types in the first step, and to focus on skill sets with sufficiently small distances between adjacent skill types in the second step. The discrete skill set has two advantages: First, the optimal tax problem involves a finite number of distinguishable downwards and upwards IC constraints, which can be added or deleted one by one to study partially relaxed problems. Second, there exist allocations in which both local IC constraints are slack for some pairs of adjacent skill types.[16] The proofs of my main results exploit both properties, and would hence not be valid with a continuous skill set as in Jacquet et al. (2013).[17]

The discrete skill set complicates one crucial step of the analysis, however. With a strictly positive distance between adjacent skill types, it is impossible to check whether a potentially optimal allocation satisfies a particular IC constraint unless strong functional form assumptions are imposed. I solve this problem by verifying incentive compatibility when the distance between adjacent skill types converges to zero. This allows me to determine unambiguously which IC constraints are binding whenever the skill set is sufficiently "dense", i.e., the distance between adjacent skill types is strictly positive but small. Hence, I study the behavior of the model at the transition between a discrete skill set and a continuous skill set, exploiting crucial

[15]A rigorous formal derivation of this result is provided in Section 6.

[16]With a continuous skill set, downwards and upwards IC constraints are collapsed into an envelope condition that is satisfied with equality in every implementable allocation.

[17]Note, however, that the qualitative result of this paper seem to remain unchanged in the limit case where the discrete skill set converges to an interval (see Lemma 26 in Appendix B.5).

advantages of both model classes.

The paper proceeds as follows. Section 2 briefly reviews the related literature. Section 3 introduces the model and the optimal tax problem. Section 4 imposes three assumptions on the primitives of the model and clarifies their implications for observable quantities. Section 5 presents the theoretical results. Section 6 explains the economic mechanism underlying these results, focusing on the trade-off between labor supply distortions at both margins. Section 7 provides numerical simulations for a version of the model that is calibrated to the US economy. Section 8 concludes. All formal proofs are provided in Appendix A.[18]

2 Related literature

Most of the previous literature on optimal non-linear income taxation focuses on two classes of models that differ in the type of costs agents face and, correspondingly, the margin at which they respond to tax changes.

First, a large number of papers follow Mirrlees (1971) by assuming that the agents face only variable costs of providing effort, which are affected by a single private parameter referred to as skill. In these models, labor supply responds to tax changes at the intensive (hours, effort) margin only. The "central result" (Hellwig 2007: 1449) of this literature is that the optimal marginal tax is strictly positive almost everywhere.[19] This result holds whenever the welfare function gives rise to a desire for redistributing resources from higher-skilled to lower-skilled agents (see, amongst others, Seade 1977, 1982, Hellwig 2007). Notably, the same results apply whether the skill set is continuous or discrete (see Stiglitz 1982 and Hellwig 2007).[20]

Second, a smaller set of papers follow Diamond (1980) by studying models in which the agents do not only differ in skills, but also in fixed costs of working. This strand of the literature was revived by Saez (2002) and a series of papers by Laroque (2005) and Choné & Laroque (2005, 2011). In their models, there are no variable costs of providing output. Hence, all agents prefer to either work at full capacity or to be unemployed: labor supply responds to tax changes at the extensive (participation) margin only. The authors find that optimal participation taxes are negative at low income levels if and only if the social planner cares almost as much

[18] Appendix B provides supplementary results and graphical illustrations.

[19] The optimal marginal income tax is only zero at the very top and, under certain conditions, at the very bottom of the income distribution.

[20] In this setting, negative marginal taxes can only be optimal if the planner has a non-standard desire to redistribute resources from low-income earners to high-income earners as in Choné & Laroque (2010) and Brett & Weymark (2017), or if the agents fail to maximize their own well-being, e.g., due to a presence bias as in Lockwood (2017).

for the low-skilled workers as for the unemployed (see Diamond 1980, Saez 2002, Choné & Laroque 2011, Christiansen 2015). While optimal marginal taxes can also be computed, they are economically irrelevant as they do not lead to distortions at the intensive margin. Again, the results do not depend on whether the skill set is continuous as in Choné & Laroque (2011) or discrete as in Christiansen (2015).

Finally, there exist a few papers that study optimal income taxation with labor supply responses at both margins. Saez (2002) strongly advocates the mixed model based on its empirical relevance and discusses how the mechanisms of this model differ from the pure intensive and the pure extensive model. He is also the first to show that negative marginal taxes at low-income levels are compatible with a standard desire for redistribution if both the optimal participation taxes and the participation elasticities of higher-income earners are sufficiently large. Unfortunately, this crucial insight does not allow to verify the optimality of an *EITC* because the optimal participation taxes are endogenous entities that depend on the redistributive concerns, the labor supply elasticities and the optimal marginal taxes themselves. Additionally, Saez (2002) provides numerical simulations of the optimal income tax, finding throughout positive marginal taxes and (sometimes) negative participation taxes at low income levels.

Mostly closely related to my paper is the one by Jacquet et al. (2013), who study optimal income taxation in a random participation model with two-dimensional heterogeneity. The main difference to my model is that they consider a type set that is continuous in both dimensions, while I study a hybrid model with a discrete set of skill types and a continuous set of fixed cost types.[21] The main result of Jacquet et al. (2013) is given by a sufficient condition for the optimality of positive marginal taxes. While this condition is expressed in terms of endogenous entities – participation elasticities and marginal social weights – as acknowledged by the authors, they also provide examples for which it is unambiguously satisfied. For example, they show that the optimal marginal taxes are positive if (a) the social planner maximizes a Rawlsian welfare function and (b) higher-skilled workers respond more elastically at the extensive margin that lower-skilled workers, in line with the empirical evidence. Moreover, they perform numerical simulations, finding that the optimal marginal tax is positive in all considered cases, while the optimal participation tax is sometimes negative (as in Saez 2002).[22]

[21] Besides, their analysis is somewhat more general in allowing for income effects in labor supply.

[22] Lorenz & Sachs (2012) complement these results by providing a condition under which optimal participation taxes are positive everywhere. Lehmann et al. (2014) and Scheuer (2014) use similar random-participation models to study optimal taxation with labor supply responses at the intensive margin and another extensive margin. In particular, the agents can choose to migrate in Lehmann et al. (2014) and change their occupation in Scheuer (2014). Although their models and research

Finally, Beaudry et al. (2009) show that an *EITC* with negative marginal taxes is always optimal in a two-dimensional screening model that deviates in several aspects from the previously discussed literature. First, the agents do not face fixed costs of working, but opportunity costs related to the possibility of generating (higher) income in an informal or black labor market. Hence, the social planner holds a desire to redistribute resources from unemployed agents (the workers in the informal sector) to formally employed agents with identical skills. Second, the planner is able to observe hours worked in the formal sector and, consequently, to condition tax payments on the wages of formally employed agents. Due to these two properties and in contrast to my model, an *EITC* with negative marginal taxes is always optimal for agents earning wages below some cutoff wage, and the optimal transfers to unemployed agents are always zero.

3 Model

The following subsection presents a two-dimensional screening model in which labor supply responds to tax changes at the intensive margin and the extensive margin. Subsection 3.2 provides formal definitions of the optimal tax problem and of labor supply distortions at both margins. Subsection 3.3 explains how the optimal allocation can be decentralized via non-linear income taxes, and Subsection 3.4 discusses the relation between social welfare functions and marginal social welfare weights.

3.1 The economy

The set of agents is given by a continuum of mass one and denoted by I, with typical element i. Agent i's consumption is denoted by c^i, his contribution to the economy's output by y^i. Agent i derives utility from consumption and suffers from the cost of providing output. This cost can be separated into a variable effort cost and a fixed cost of participating in the labor market. Formally, individual preferences can be represented by the following utility function:[23]

$$u(c^i, y^i; \omega^i, \delta^i) = c^i - h\left(y^i, \omega^i\right) - \mathbb{1}_{y^i>0}\, \delta^i. \tag{1}$$

The fixed cost of participating in the labor market is given by an individual parameter $\delta^i \in \Delta$, which I refer to as agent i's *fixed cost type*. The variable effort cost of providing output is measured by function h. It depends on the output level y^i

questions differ from those in my paper, the mechanisms at work are similar.

[23]I comment on the implications of the functional form imposed by (1) below.

and an individual parameter $\omega^i \in \Omega$, which I refer to as i's *skill type*. Absolute and marginals cost of providing output are decreasing in this parameter, so that $h_\omega(y^i, \omega^i) < 0$ and $h_{y\omega}(y^i, \omega^i) < 0$ for all $y^i > 0$ and $\omega^i \in \Omega$. Moreover, h is strictly increasing and strictly convex in y^i, so that $h_y(y^i, \omega^i) > 0$ and $h_{yy}(y^i, \omega^i) > 0$ for all $y^i > 0$ and $\omega^i \in \Omega$. Finally, h is assumed to satisfy $h(0, \omega^i) = 0$, $h_{yy\omega}(y, \omega) \leq 0$ for all $y^i > 0$ and $\omega^i \in \Omega$ as well as the Inada conditions $\lim_{y \to 0} h_y(y^i, \omega^i) = 0$ and $\lim_{y \to \infty} h_y(y^i, \omega^i) = \infty$ for all $\omega^i \in \Omega$.

Agent i is privately informed about his skill type ω^i and his fixed cost type δ^i. The skill set Ω is given by a finite ordered set $\{\omega_1, \omega_2, \ldots, \omega_n\}$ with $\omega_{j+1}/\omega_j \geq 1 + \varepsilon$ for all $j \in \{1, 2, \ldots, n-1\}$ and some $\varepsilon > 0$. The set of fixed costs Δ is given by a closed interval with lower endpoint $\underline{\delta}$ and upper endpoint $\bar{\delta}$, assumed to satisfy

$$\underline{\delta} < \max_{y>0} \; y - h\left(y, \omega_1\right) \;, \quad \bar{\delta} > \max_{y>0} \; y - h\left(y, \omega_n\right) \;. \tag{2}$$

Under *laissez-faire*, agents with fixed cost type $\underline{\delta}$ and any skill type $\omega \in \Omega$ would thus provide positive output, while agents with fixed cost type $\bar{\delta}$ and any skill type $\omega \in \Omega$ would provide zero output.[24] As will become clear below, the combination of a discrete set of skills and a continuous set of fixed costs helps to explain the interaction between labor supply distortions at both margins.

The joint cross-section distribution of the pair (ω^i, δ^i) in the population at large is commonly known and denoted by $K : \Omega \times \Delta \to [0, 1]$. The share of agents with skill type ω_j, which I henceforth refer to as skill group j, is given by the number $f_j > 0$ for any $j \in j$. The distribution function of fixed cost types in any skill group $j \in J$ is twice continuously differentiable and denoted by G_j. The corresponding density function g_j is bounded from below by some number $\underline{g} > 0$ for all $\delta \in \Delta$ and there exists some closed subset of Δ on which g_j is weakly decreasing.

3.2 The optimal tax problem

I use a mechanism design approach to solve for the optimal non-linear income tax. Thus, I study the problem to maximize a social welfare function (to be defined below) over the set of implementable, i.e., feasible and incentive-compatible, allocations. An allocation is given by two functions $c : \Omega \times \Delta \to \mathbb{R}$ and $y : \Omega \times \Delta \to \mathbb{R}_0^+$ that specify the consumption and output levels for all types in $\Omega \times \Delta$. It is *feasible* if overall consumption does not exceed overall output, i.e.,

$$\int_{\Omega \times \Delta} c(\omega, \delta) dK(\omega, \delta) \leq \int_{\Omega \times \Delta} y(\omega, \delta) dK(\omega, \delta), \tag{3}$$

[24]This ensures that tax changes may affect the participation decisions of agents in all skill groups.

and *incentive-compatible* if

$$u\left(c(\omega,\delta),y(\omega,\delta);\omega,\delta\right)\geq u\left(c(\omega',\delta'),y(\omega',\delta');\omega,\delta\right) \tag{4}$$

for all types (ω,δ) and (ω',δ') in $\Omega\times\Delta$.

This maximization problem can be simplified considerably by focusing on the set of allocations that are implementable as well as (second-best) Pareto efficient.[25] In particular, Lemma 1 shows that any such allocation involves pooling by $n+1$ sets of different types.

Lemma 1. *Every allocation* (c,y) *that is implementable and Pareto efficient (in the set of implementable allocations) is characterized by two vectors* $(y_j)_{j=1}^n$*,* $(c_j)_{j=0}^n$ *such that,*

- *for each* $j\in J=\{1,2,\ldots,n\}$*, all agents with skill type* ω_j *and fixed cost type* $\delta\leq\delta_j:=c_j-h(y_j,\omega_j)-c_0$ *receive bundle* (c_j,y_j)*, and*
- *all other agents receive bundle* $(c_0,0)$*.*

By Lemma 1, any implementable and (second-best) Pareto efficient allocation satisfies two properties. First, all agents with skill type $\omega_j\in\Omega$ and fixed cost types below the (endogenous) participation threshold δ_j provide the same output level $y_j>0$ and receive the same consumption level $c_j>0$. Second, all other agents provide zero output and receive the same consumption level c_0. Both properties are driven by the additive separability of the fixed cost component δ in utility function (1), which follows the random participation approach by Rochet & Stole (2002).[26]

Below, I will investigate the labor supply distortions in the optimal allocation. The characterization of these distortions is based on the following thought experiment, which I illustrate in Figures 4 and 5 in Appendix B.6. Consider an initial allocation in which agent i's bundle is given by $(c^i,y^i)\geq 0$. Now consider providing agent i with a different bundle $(\tilde{c},\tilde{y})\geq 0$ such that $\tilde{y}-y^i=\tilde{c}-c^i\neq 0$. The set of these potential deviations is given by a straight line through (c^i,y^i) with slope equal to 1, the economy's *marginal rate of transformation* between consumption and output. Agent i's labor supply is said to be distorted if there is a bundle $(\tilde{c},\tilde{y})$ on this line that i strictly prefers to (c^i,y^i).

[25] For all welfare functions considered below, the welfare-maximizing allocation is ensured to be (second-best) Pareto efficient by standard reasons.

[26] The first property follows because, conditional on working, an agent's preferences over any set of bundles depend only on his skill type. Hence, all workers in skill group $j\in J$ must receive the same gross payoff $c-h(y,\omega)$. Pareto efficiency requires that they also receive the same bundle (c_j,y_j). The second property follows because, conditional on not working, an agent's payoff is independent of his type.

First, it might be possible to increase i's utility through a marginal deviation from (c^i, y^i). This will be the case if and only if i's marginal rate of substitution, $h_y(y^i, \omega^i)$, differs from 1. If $h_y(y^i, \omega^i) < 1$, i would strictly prefer an output-increasing deviation. Then, i's labor supply is said to be *downwards distorted at the intensive margin*. Correspondingly, if $h_y(y^i, \omega^i) > 1$, i would strictly prefer an output-decreasing deviation, and i's labor supply is said to be *upwards distorted at the intensive margin*.

Second, it might be possible to increase i's utility through a large deviation from (c^i, y^i) that changes his participation status (from zero output to positive output or vice versa). Consider an initial allocation with $y^i = 0$ and the deviation to bundle $(c^i + \tilde{y}, \tilde{y})$ for some $\tilde{y} > 0$. Agent i would be strictly better off with the new bundle than with his initial bundle if and only if i's total costs of providing output $\tilde{y}$ are below the additional utility from consuming $\tilde{y}$, $h(\tilde{y}, \omega^i) + \delta^i < \tilde{y}$. Hence, i's labor supply is said to be *downwards distorted at the extensive margin* if both $y^i = 0$ and $\delta^i < \delta^*(\omega^i) := \max_{y>0} y - h(y, \omega^i)$.

Correspondingly, consider an initial allocation with $y^i > 0$. Agent i would be strictly better off with bundle $(c^i - y^i, 0)$ than with his initial bundle if and only if i's total costs of providing output y^i exceed the utility from consuming y^i, $h(y^i, \omega^i) + \delta^i > y^i$. Hence, i's labor supply is said to be *upwards distorted at the extensive margin* if both $y^i > 0$ and $\delta^i > y^i - h(y^i, \omega^i)$.

By Lemma 1, it is possible to characterize the labor supply distortions for the agents in each skill group $j \in J$ simultaneously. In particular, labor supply in skill group j is said to be distorted at the intensive margin if the marginal rate of substitution $h(y_j, \omega_j)$ differs from one for all *working agents* with skill type ω_j. Similarly, labor supply in skill group j is said to be distorted at the extensive margin if the skill-specific *participation threshold* δ_j is either located below $\delta^*(\omega_j)$ (downward distortion) or above $y_j - h(y_j, \omega_j)$ (upward distortion).

Finally, an allocation is said to be efficient if it does not involve labor supply distortions at any margin. Using Lemma 1, the efficiency losses in all other allocations can be evaluated based on the implied *deadweight loss*, defined as

$$\begin{aligned} DWL(c, y) \;:=\; & \sum_{j=1}^{n} f_j \int_{\underline{\delta}}^{\delta^*(\omega_j)} g_j(\delta)\,[\delta^*(\omega_j) - \delta]\,d\delta \\ & - \sum_{j=1}^{n} f_j \int_{\underline{\delta}}^{\delta_j} g_j(\delta)\,[y_j - h(y_j, \omega_j) - \delta]\,d\delta\,, \end{aligned} \tag{5}$$

where $\delta_j = c_j - h(y_j, \omega_j) - c_0$ as derived in Lemma 1.[27]

[27] In Appendix B.7, I explain the derivation of (5) and show that the overall deadweight loss can

3.3 Decentralization

In Section 5 below, I provide the main results of this paper by characterizing the labor supply distortions in the optimal allocation at both margins. As I demonstrate in the following, these results can straightforwardly be translated into statements about the signs of the optimal marginal tax and the optimal participation tax.

Consider the class of social choice functions that are decentralized through non-linear income tax schedules, mapping output levels into tax payments. I denote by (c_T, y_T) the social choice function that is decentralized by tax function T. The literature also refers to y_T as pre-tax income, and to c_T as post-tax income. Tax function T is admissible if the tax revenue is non-negative, $\int_{\Omega\times\Delta} T\left[y_T(\omega,\delta)\right] dK(\omega,\delta) \geq 0$.

The problem of an agent with type (ω, δ) is to choose income y in order to maximize $U(c, y; \omega, \delta)$, subject to the individual budget constraint $c = y - T(y)$. To simplify the exposition, assume that T is continuously differentiable. Then, the solution to this program is given by

$$y_T(\omega,\delta) = \begin{cases} y_T^*(\omega) & \text{if} \quad \delta \leq \delta_T(\omega) \\ 0 & \text{if} \quad \delta > \delta_T(\omega), \end{cases} \tag{6}$$

where $y_T^*(\omega)$ is implicitly defined by

$$1 - T'\left[y_T^*(\omega)\right] = h_y\left(y_T^*(\omega), \omega\right), \tag{7}$$

and $\delta_T(\omega) := y_T^*(\omega) - h\left(y_T^*(\omega), \omega\right) - \left[T\left(y_T^*(\omega)\right) - T(0)\right]$.

The case distinction again results due to the additive separability imposed by (1). Conditional on working, the agent's optimal income $y_T^*(\omega)$ is defined by the first-order condition (7), which involves only his skill type ω and the marginal tax T'.[28] Consequently, participating in the labor market can increase the agent's utility at most by $\delta_T(\omega)$, the net gain from increased consumption and increased effort costs. If his fixed cost δ is lower than $\delta_T(\omega)$, the conditional optimum $y_T^*(\omega)$ hence maximizes the agent's utility. If δ instead exceeds $\delta_T(\omega)$, the agent prefers zero income to any positive income $y > 0$. Note that the fixed cost type δ and the participation tax $T(y) - T(0)$ affect only the extensive decision whether or not to work at all, but not the intensive decision how much to work.

Consider an implementable and Pareto-efficient allocation (c, y), given by the consumption-output bundles of the workers in each skill group $j \in J$ and the unem-

be decomposed in efficiency losses from distortions at both margins.

[28] If T is strictly convex, the first-order condition (7) may be satisfied at several income levels. If T is not continuously differentiable, $y_T^*(\omega)$ might be located at a kink of T and fail to satisfy (7). For the sake of clarity, I abstract from both complications here.

ployed (see Lemma 1). By the taxation principle, any such allocation can be decentralized by a non-linear tax schedule T so that $(c_T, y_T) = (c, y)$. The required properties of tax T depend on the labor supply distortions in allocation (c, y).

First, as usual, if labor supply in skill group j is downwards distorted at the intensive margin, the *marginal tax* at income level $y_T^*(\omega_j) = y_j$ must be strictly positive. Correspondingly, if labor supply in skill group j is upwards distorted at the intensive margin, the *marginal tax* at income level $y_T^*(\omega_j) = y_j$ must be strictly negative. Intuitively, a positive (negative) *marginal tax* is required to ensure that each agent with skill type ω_j provides lower (higher) output than efficient.[29]

Second, the required sign of the participation tax has to correspond to the extensive-margin distortions in (c, y). In particular, if labor supply in skill group j is downwards distorted at the extensive margin, $\delta_j < \delta^*(\omega_j)$, the *participation tax* $T(y) - T(0)$ must be strictly positive at the income level that maximizes $y - h(y, \omega_j)$.[30] Intuitively, a positive *participation tax* is required to ensure that the agent chooses to inefficiently stay out of the labor market. Correspondingly, if labor supply in skill group j is upwards distorted at the extensive margin, the *participation tax* at income level $y_T^*(\omega) = y_j$ must be strictly negative.[31]

3.4 The social welfare function

In optimal tax theory, social welfare is usually taken to be an increasing function of individual utilities that gives rise to a "desire for redistribution" (Hellwig 2007). A standard assumption is that the social objective can be expressed as

$$\int_{\Omega \times \Delta} \gamma(\omega, \delta)\ \Psi\left(c(\omega, \delta) - h\left[y(\omega, \delta), \omega\right] - \mathbb{1}_{y(\omega,\delta)>0}\ \delta\right) dK(\omega, \delta), \tag{8}$$

where the transformation $\Psi : \mathbb{R} \to \mathbb{R}$ is strictly increasing and weakly concave, and the weighting function $\gamma : \Omega \times \Delta \to \mathbb{R}_+$ is weakly decreasing in ω and weakly increasing in δ. The desire for redistribution is either introduced through transformation Ψ or through type-dependent weights γ; with quasi-linear preferences, it would not be present if welfare were given by the unweighted sum of individual utilities.

In an economy with heterogeneity in skills only, the concavity of Ψ and the monotonicity of γ ensure that society has a standard concern for redistribution

[29] By definition, labor supply in skill group j is downwards distorted at the intensive margin if the marginal rate of substitution at y_j is below 1. The marginal tax must hence satisfy $T'\left(y_T^*(\omega_j)\right) = 1 - h_y\left(y_T^*(\omega_j), \omega_j\right) > 0$ to ensure that the workers in skill group j choose income $y_T^*(\omega_j) = y_j$.

[30] As shown above, an agent self-selects zero income if his fixed cost type δ exceeds $y - h(y, \omega) - [T(y) - T(0)]$, the utility gain of participation, for all positive income levels.

[31] To the best of my knowledge, this correspondence has never been shown rigorously in the previous literature.

from higher-income earners to lower-income earners. In the economy with two-dimensional heterogeneity considered here, the same assumptions fail to pin down the direction of desirable redistribution uniquely.

To see this potential ambiguity, consider the case where Ψ is strictly concave while γ is constant over all types in $\Omega \times \Delta$. Hence, society desires to redistribute resources from types enjoying high levels of utility to types with lower levels of utility. In the economy with two-dimensional heterogeneity, an agent's utility is increasing in his skill type and decreasing in his fixed cost type. Consequently, a low-skilled agents with low fixed costs may enjoy a higher utility than a high-skilled worker with high fixed costs. By Lemma 1, however, the social planner can only redistribute resources between the $n+1$ groups of agents who provide different output levels: the group of unemployed workers and the groups of workers with skill type ω_j for any $j \in J$. Hence, the concavity of Ψ is neither a necessary nor a sufficient condition for a social desire to redistribute resources from higher-income earners to lower-income earners.[32] Instead, the direction of desired redistribution depends on the properties of Ψ (and γ) and on the joint type distribution K.

To deal with this complication, I express the social concerns for redistribution by the average marginal welfare weights associated to the $n+1$ groups mentioned above, evaluated at the welfare-maximizing allocation. For any $j \in J$, I define the social weight $\bar{\alpha}_j$ as the marginal welfare effect of increasing c_j, the consumption level enjoyed by all workers with skill type ω_j,

$$\bar{\alpha}_j \coloneqq E_\delta \left[\gamma(\omega_j, \delta)\ \Psi'\left(c_j - h\left(y_j, \omega_j\right) - \delta\right) \mid \delta \leq \delta_j\right] \ . \tag{9}$$

Correspondingly, I define the social weight $\bar{\alpha}_0$ of unemployed agents as the marginal welfare effect of increasing c_0, the consumption level of all unemployed agents,

$$\bar{\alpha}_0 \coloneqq E_{\omega_j,\delta} \left[\gamma(\omega_j, \delta)\ \Psi'\left(c_0\right) \mid \delta < \delta_j, j \in J\right] \ . \tag{10}$$

The average weight across the population is given by

$$\bar{\alpha}_M \coloneqq \int_{\Omega\times\Delta} \left[\left(\bar{\alpha}_j - \bar{\alpha}_0\right) \mathbb{1}_{y(\omega,\delta)>0} \ + \bar{\alpha}_0\right] dK(\omega_j, \delta). \tag{11}$$

In the remainder of this paper, I restrict my attention to welfare functions that give rise to monotonically decreasing social weight sequences, which seems to be most natural and economically most relevant case.[33] Importantly, this restriction

[32] Similarly, the assumption that γ is decreasing in ω and increasing in δ does not ensure a concern for redistribution from higher-income earners to lower-income earners.

[33] In Appendix B.8, I provide conditions on Ψ, γ and K that jointly ensure that the social weight

simplifies the comparison of my results with those in the standard Mirrlees framework: If the optimal tax has non-standard properties, they cannot be driven by a non-monotonicity in the social weights as in, e.g., Choné & Laroque (2010), but must be related to the interaction of labor supply responses at the intensive and extensive margins.

To simplify the exposition, I henceforth treat the group-specific social weights as if they were exogenous objects and denote them by $\alpha = (\alpha_0, \alpha_1, \ldots, \alpha_n)$. Without loss of generality, I focus on the weight sequences that are normalized to have an average weight of 1. I denote the set of strictly decreasing, normalized weight sequences by $\mathcal{A}$.

4 Assumptions

In Subsection 4.1, I impose a set of assumptions on the joint type distribution and the effort cost function, i.e., the primitives of the model. As shown in Subsection 4.2, these assumptions ensure consistency of the model with the empirically observed patterns of labor supply responses. The relevance of each assumption for the results of this paper is discussed in the subsequent sections.

4.1 Assumptions on primitives

The first two conditions impose restrictions on the joint type distributions, expressed in terms of the hazard rates of fixed costs distributions. Fix a skill type ω_j. Recall that G_j denotes the *cdf* of the distribution of fixed cost types in the group of agents with this skill type. The hazard rate of this *cdf* is given by $A_j(\delta) \coloneqq \frac{g_j(\delta)}{G_j(\delta)}$.

Condition 1. *The joint type distribution has the following properties:*

(i) For each $j \in J$, $A_j(\delta)$ is strictly decreasing in δ.

(ii) For each $j \in J_{-n} \coloneqq J \setminus \{n\}$ and $\delta \in \Delta$, $A_j(\delta) \geq A_{j+1}(\delta)$.

Condition 1 requires the hazard rates of G_j to be decreasing along both type dimensions.[34] Part (i) is a standard monotone hazard rate condition that is satisfied whenever the conditional fixed-cost distributions are log-concave. Part (ii) rules out a specific type of positive joint variation between skill types and fixed cost types.

sequence $\bar{\alpha}$ is monotonically decreasing. Additionally, I provide an example in which $\bar{\alpha}$ is locally increasing although Ψ is strictly concave.

[34]Note that the same assumption is used by Choné & Laroque (2011) and Jacquet et al. (2013). Scheuer (2014) imposes a similar but stricter assumption on the joint type distribution.

For some distribution functions, it is identical to the assumption that G_j weakly dominates G_{j+1} in the sense of first-order stochastic dominance.[35]

The second condition compares the previously defined *cdf* hazard rate $A_j(\delta)$ with the hazard rate of the corresponding *pdf* in the same skill group j. I denote this *pdf* hazard rate by $a_j := \frac{g_j'(\delta)}{g_j(\delta)}$.

Condition 2. *The joint type distribution has the following properties:*

(i) For each $j \in J$, $a_j(\delta)$ is weakly decreasing in δ with $\frac{da_j(\delta)}{d\delta} \in \left[2\,\frac{dA_j(\delta)}{d\delta}, 0\right]$.

(ii) For each $j \in J_{-n}$ and $\delta \in \Delta$, $0 \leq a_j(\delta) - a_{j+1}(\delta) \leq 2\,[A_j(\delta) - A_{j+1}(\delta)]$.

Condition 2 imposes two novel conditions that have not been used in the literature before.[36] They require that the *pdf* hazard rate a_j varies across both type dimensions in the same direction as the *cdf* hazard rate A_j, but at a sufficiently small rate compared to the latter. Both parts of Condition 2 are satisfied, e.g., if the fixed cost distributions are uniform or logistic for all skill groups. Moreover, part (ii) trivially holds whenever skill types and fixed cost types are independently distributed.[37]

The third condition imposes mild restrictions on the effort cost function h.

Condition 3. *There are two numbers $\mu_1 \in (0, \infty)$ and $\mu_2 \in (0, \infty)$ such that, for each $y > 0$ and $\omega > \omega_1$, the effort cost function h satisfies*

(i) $\frac{1}{y}\frac{h_y(y,\omega)}{h_{yy}(y,\omega)} \leq \mu_1$, and

(ii) $-\frac{\omega}{y}\frac{h_{y\omega}(y,\omega)}{h_{yy}(y,\omega)} \geq \mu_2$.

Condition 3 is satisfied for all commonly used functional forms, including the class of functions given by $h(y,\omega) = \frac{1}{1+1/\sigma}\left(\frac{y}{\omega}\right)^{1+1/\sigma}$ for any $\sigma \in (0, \infty)$.

4.2 Implications for labor supply elasticities

Condition 1 puts a restriction on the relative responses at the extensive margin in different skill groups. It proves helpful to measure these responses by the semi-elasticity η_j of participation in each skill group j, formally defined by

$$\eta_j(c,y) := \frac{\partial G_j\left(c_j - h\left(y_j, \omega_j\right) - c_0\right)}{\partial c_j} \frac{1}{G_j\left(c_j - h\left(y_j, \omega_j\right) - c_0\right)} = A_j(\delta_j) \qquad (12)$$

[35] In general, Condition 1 (ii) is neither implying nor implied by first-order stochastic dominance.

[36] For the discrete set of skills studied here, Condition 2 gives rise to a monotonicity result that simplifies the following analysis. With a continuous set of skills as in most previous papers, this monotonicity result would come for free.

[37] In Appendix B.9, I provide a set of commonly used distribution functions for which Conditions 1 and 2 are satisfied.

for each $j \in J$.[38] Condition 1 ensures consistency with the empirical findings that low-skill workers respond more elastically at the extensive margin than high-skill workers (see, e.g., Juhn et al. 1991, 2002 and Meghir & Phillips 2010).

Lemma 2. *For each $j \in J_{-n}$, skill group j has a strictly larger semi-elasticity of participation than skill group $j+1$, $\eta_j(c, y) > \eta_{j+1}(c, y)$, in every implementable allocation.*

As can be seen from equation (12), the semi-elasticities of participation are endogenous objects that vary with allocation (c, y). In particular, Condition 1 implies that a uniform increase in the consumption levels of workers in skill groups j and $j+1$ leads to a reduction in the semi-elasticities η_j and η_{j+1}. The effect on the ratio of both semi-elasticities $\hat{\eta}_{j,j+1} := \eta_j/\eta_{j+1}$ can go in any direction and have any magnitude, however. Condition 2 rules out erratic fluctuations of $\hat{\eta}_{j,j+1}$ by imposing bounds on the semi-elasticity $\varepsilon_{\hat{\eta},c}$ of this ratio with respect to such a uniform transfer.[39]

Lemma 3. *For each $j \in J_{-n}$, the semi-elasticity of the relative participation responses $\hat{\eta}_{j,j+1} := \eta_j/\eta_{j+1}$ with respect to uniform transfers satisfies*

$$|\varepsilon_{\hat{\eta},c}(c, y)| < \eta_j(c, y) - \eta_{j+1}(c, y) \ .$$

While $\varepsilon_{\hat{\eta},c}$ is in principle an observable quantity, I am not aware of any empirical results on its sign or magnitude. Under Condition 2, it may be positive or negative, but has to be sufficiently small in absolute terms. Given the lack of empirical evidence, this seems to be a reasonably weak assumption.

Finally, Condition 3 puts mild restrictions on the labor supply responses at the intensive margin. Effectively, it ensures that labor supply does not respond in a degenerate way to tax changes.

Lemma 4. *For each $j \in J$, the elasticity of income with respect to*

(i) the retention rate $1-T'(y)$ is bounded from above by some number $\mu_1 \in (0, \infty)$;

(ii) the skill level ω is bounded from below by some number $\mu_2 \in (0, \infty)$.

[38] More precisely, η_j represents the semi-elasticity of the skill-specific participation share $G_j(\delta_j)$ with respect to the net-of-tax income $c_j = y_j - T(y_j)$ faced by the workers with skill type ω_j. It indicates by how much percent the participation share in skill group j increases if c_j is increased by one unit.

[39] Formally, I define the semi-elasticity of the relative participation responses $\hat{\eta}_{j,j+1}$ as $\varepsilon_{\hat{\eta},c}(c, y) := \frac{\partial \hat{\eta}_{j,j+1}(\delta_j, \delta_{j+1}, 0)}{\partial c'} \frac{1}{\hat{\eta}_{j,j+1}(\delta_j, \delta_{j+1}, 0)}$, where $\hat{\eta}_j(\delta_j, \delta_{j+1}, c') = \frac{g_j(\delta_j + c')}{G_j(\delta_j + c')} \frac{G_{j+1}(\delta_{j+1} + c')}{g_{j+1}(\delta_{j+1} + c')}$.

Condition 3 can hence be regarded as a weak regularity condition that guarantees a minimal degree of consistency with the empirical evidence. It is worth noting that the results of this paper would be unaffected if I would directly assume the before-mentioned properties of labor supply elasticities to hold, instead of imposing assumptions on the primitives in the first place.

5 Results

In the following, I present the formal results of this paper. I start by investigating a relaxed version of the optimal tax problem, for which I derive two preliminary results of crucial importance. Then, I provide two results on the optimality of upwards distortions at both margins in the solution to the non-relaxed problem. As explained in Subsection 3.3, these results can easily be translated into results on the optimality of negative marginal taxes and participation taxes. The economic mechanism behind these results will be explained in the following section.

5.1 Preliminary results

As Jacquet et al. (2013), I start by studying a relaxed version of the optimal tax problem that ignores the incentive compatibility (IC) constraints between working agents with different skill types. More precisely, I study the problem of maximizing social welfare (8) subject to the feasibility constraint (3) and the subset of IC constraints, first, between all agents with identical skill types,

$$u\left(c(\omega,\delta),y(\omega,\delta);\omega,\delta\right) \geq u\left(c(\omega,\delta'),y(\omega,\delta');\omega,\delta\right) \tag{13}$$

for each $\omega \in \Omega$ and $\delta,\delta' \in \Delta$, and second, between all unemployed agents,

$$c(\omega,\delta) \geq c(\omega',\delta') \tag{14}$$

for each pair (ω,δ) and (ω',δ') in $\Omega \times \Delta$ such that $y(\omega,\delta) = y(\omega',\delta') = 0$.

Lemma 1 continues to apply to the set of allocations satisfying this reduced set of IC constraints. Hence, the solution to the relaxed problem involves, first, pooling by all unemployed agents and, second, pooling by all working agents with the same skill type. Moreover, an agent with skill type ω_j provides positive output if and only if his fixed cost type is below the skill-specific threshold δ_j. As a result, the solution to the relaxed problem for the social weight sequence α can be denoted by the vectors $\left(c_j^{\alpha R}\right)_{j=0}^n$, $\left(y_j^{\alpha R}\right)_{j=1}^n$ and $\left(\delta_j^{\alpha R}\right)_{j=1}^n$, where $\delta_j^{\alpha R} = c_j^{\alpha R} - h(y_j^{\alpha R},\omega_j) - c_0^{\alpha R}$.

Lemma 5. *There is a number $\chi \in (1,2]$ such that, if $\alpha_j \in [0,\chi)$ for all $j \in J$, the relaxed problem has a unique solution that satisfies*

$$h_y(y_j^{\alpha R}, \omega_j) = 1 \qquad \forall\, j \in J\ , \tag{15}$$

$$\delta_j^{\alpha R} - \delta^*(\omega_j) = \frac{\alpha_j - 1}{A_j(\delta_j^{\alpha R})} \qquad \forall\, j \in J\ , \text{ and} \tag{16}$$

$$c_0^{\alpha R} = \sum_{j=1}^{n} f_j G_j(\delta_j^{\alpha R}) \left(\delta^*(\omega_j) - \delta_j^{\alpha R}\right)\ . \tag{17}$$

First, Lemma 5 indicates that the relaxed problem is well-behaved if the social weights of all groups of workers are below some threshold $\chi \in (1,2]$. For higher social weights, in contrast, an allocation may satisfy the first-order conditions, but violate a second-order condition. This is a well-known problem for models with labor supply responses at the extensive margin.[40] The resulting technical complications are not directly related to the research question of this paper. Hence, I proceed by restricting my attention to a subset of social weight sequences for which the existence of a well-behaved solution is ensured. In particular, I define the set $\mathcal{A}^\chi \subset \mathcal{A}$ as the subset of strictly decreasing weight sequences for which each element α_j is below the threshold χ. All results in the remainder of this paper will be derived for weight sequences in $\mathcal{A}^\chi$.

Second, equations (15) and (16) characterize the labor supply distortions in the solution to the relaxed problem. At the intensive margin, optimal output $y^{\alpha R}$ is undistorted in all skill groups. This may not come as a surprise to the reader, because the IC constraints between different skill groups have not been taken into account in the relaxed problem.[41] At the extensive margin, in contrast, optimal output $y^{\alpha R}$ can be distorted in both directions. In particular, labor supply is distorted downwards at the extensive margin in each skill group with a social weight below the average weight of 1, and distorted upwards in each skill group with a weight above 1. With decreasing social weights, upwards distortions can only be optimal at the bottom of the skill distribution, consequently.

These results give rise to the crucial questions whether the introduction of the previously omitted IC constraints, first, leads to downwards, upwards or no distortions at the intensive margin, and second, changes this simple pattern of distortions at the extensive margin. In particular, upwards distortions at the intensive margin can only be expected to be optimal if the solution to the relaxed problem violates

[40] Formally, the Lagrangian can become strictly convex in c_j for $\alpha_j > \chi$. In this case, the welfare function may have multiple local extrema (see also discussion in Choné & Laroque 2011).

[41] In the classical model by Mirrlees (1971), distortions at the intensive margin are optimal because they allow to relax the binding (downwards) IC constraints between adjacent skill types.

the upward IC constraints,

$$c_j^{\alpha R} - h\left(y_j^{\alpha R}, \omega_j\right) \geq c_{j+1}^{\alpha R} - h\left(y_{j+1}^{\alpha R}, \omega_j\right) \ , \tag{18}$$

for some pairs of adjacent skill types (ω_j, ω_{j+1}). In an intensive-margin model à la Mirrlees (1971), this is impossible as long as social weights are decreasing, $\alpha_j > \alpha_{j+1}$. In the present model with labor supply responses at both margins, the answer to this question is more subtle.

In the relaxed problem's solution, the workers in skill group $k \in \{j, j+1\}$ receive a bundle (c_k, y_k) that satisfies $c_k^{\alpha R} - h\left(y_k^{\alpha R}, \omega_k\right) = \delta_k^{\alpha R} + c_0^{\alpha R}$. Using the implicit definitions of $\delta_j^{\alpha R}$ and $\delta_{j+1}^{\alpha R}$ in equation (16) and rearranging terms, one finds that the upward IC constraint between skill groups j and $j+1$ is violated if and only if

$$\frac{\alpha_{j+1} - 1}{\eta_{j+1}\left(c^{\alpha R}, y^{\alpha R}\right)} > \frac{\alpha_j - 1}{\eta_j\left(c^{\alpha R}, y^{\alpha R}\right)} + \mathcal{B}_j \ , \tag{19}$$

where the term $\mathcal{B}_j$ is strictly positive and depends on the distance between ω_j and ω_{j+1} and on $\varepsilon_{y,1-T'}$ and $\varepsilon_{y,\omega}$, the elasticities of income with respect to the retention rate and the skill level.[42]

By equation (19), it mainly depends on two statistics whether the upward IC constraint between skill types j and $j+1$ is satisfied or violated: the social weights α_j and α_{j+1} and the semi-elasticities of participation η_j and η_{j+1}.[43] Recall that α_j is larger than α_{j+1} for all considered weight sequences, and that η_j is larger than η_{j+1} by Lemma 2. Hence, equation (19) provides two important insights. First, the upward IC constraint can only be violated if both social weights α_j and α_{j+1} are above the population average of 1 and relatively close to each other. Second, if this condition is met, then an increase in the relative participation responses η_j/η_{j+1} makes the upward IC constraint more likely to be violated. Both insights are limited by the fact that the semi-elasticities η_j and η_{j+1} are endogenous quantities that depend on the social weights α_j and α_{j+1}. To proceed, I can however exploit that equation (19) implicitly defines a function $\beta_j^U : [0, \chi) \to \mathbb{R}$ such that the upward IC constraint is violated if and only if $\alpha_{j+1} > \beta_j^U(\alpha_j)$.

Similarly, I can use equation (16) to define a function $\beta_j^D : [0, \chi) \to \mathbb{R}$ such that

[42] In Appendix B.3, I formally derive the term $\mathcal{B}_j$ and show that it can be approximated by $0.5 y_j^{\alpha R} h_y[(y_j^{\alpha R} + y_{j+1}^{\alpha R})/2, \omega_j] \varepsilon_{y,\omega}^2 \varepsilon_{y,1-T'}^{-1} \left(\omega_{j+1}/\omega_j - 1\right)^2$, where the elasticities are evaluated at $(y_j^{\alpha R}, \omega_j)$.

[43] Jacquet et al. (2013) provide a similar condition for the model with a continuous skill set. Their condition does not contain the term $\mathcal{B}_j$, which vanishes for $\omega_{j+1}/\omega_j \to 1$.

the downward IC constraint

$$c_{j+1}^{\alpha R} - h\left(y_{j+1}^{\alpha R}, \omega_{j+1}\right) \geq c_j^{\alpha R} - h\left(y_j^{\alpha R}, \omega_{j+1}\right) \ , \tag{20}$$

is violated if and only if $\alpha_{j+1} < \beta_j^D(\alpha_j)$.[44] For both functions, there exist no closed-form expressions. Nevertheless, I can use them to identify the pairs of social weights (α_j, α_{j+1}) for which each local IC constraint is violated in allocation $(c^{\alpha R}, y^{\alpha R})$.

Lemma 6. *For each $j \in J_{-n}$, the functions β_j^D and β_j^U are continuously differentiable, strictly increasing and satisfy $0 < \beta_j^D(x) < \beta_j^U(x) < \chi$ for any $x \in [0, \chi)$. For each $j \in J_{-n}$, there is a number $a_j > 1$ such that, if $\omega_{j+1}/\omega_j \in (1, a_j)$,*

(a) $\beta_j^D(x) < x$ if and only if x is above a unique number $\underline{\beta}_j \in (0, 1)$, and

(b) $\beta_j^U(x) < x$ if and only if x is above a unique number $\bar{\beta}_j \in (1, \chi)$.

Lemma 6 provides conditions under which the relaxed problem's solution, first, violates the downward IC constraint, second, satisfies both IC constraints or, third, violates the upward IC constraint between the workers in skill groups j and $j+1$, depending only on the social weights α_j and α_{j+1}. If the relative distance between skill levels ω_j and ω_{j+1} is sufficiently small, each of these three cases arises for some pair of social weights with $\alpha_j > \alpha_{j+1}$.[45]

Figure 1 illustrates the formal statements in Lemma 6 to make them more easily accessible. The shaded area below the 45° line comprises all possible combinations of the social weights α_j and $\alpha_{j+1} < \alpha_j$ in the relevant set $\mathcal{A}^\chi$. Additionally, Figure 1 contains two ascending graphs corresponding to the functions β_j^D and β_j^U in Lemma 6. Each graph crosses the 45° line exactly once, i.e., each function has a unique fixed point. The fixed point $\underline{\beta}_j$ of function β_j^D is located between 0 and the average weight 1. The fixed point $\bar{\beta}_j$ of function β_j^U is located between the average weight 1 and the upper threshold χ. By Lemma 6, the functions β_j^D and β_j^U partition the relevant set of weights $\mathcal{A}$ into three regions.

For each pair (α_j, α_{j+1}) in region I, the relaxed problem's solution violates the downward IC constraint, i.e., higher-skilled workers consider their bundle $(c_{j+1}^{\alpha R}, y_{j+1}^{\alpha R})$ less attractive than the bundle of the lower-skilled workers, (c_j, y_j). As can be seen from Figure 1, this constellation results if either both social weights are low (below $\underline{\beta}$) or if the difference between the social weights α_j and α_{j+1} is large. In the first

[44] For completeness, Appendix B.3 provides a condition that allows to determine whether the downward IC constraint is satisfied or violated based on labor supply elasticities and social weights (corresponding to equation 19).

[45] If the relative distance between ω_j and ω_{j+1} exceeds the bound a_j, the solution of the relaxed problem may satisfy both local IC constraints (or one of them) for all α_j and α_{j+1} in $(0, \chi)$.

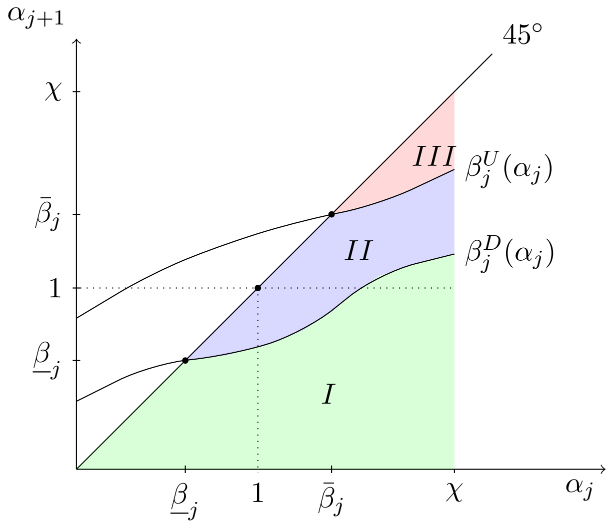

Figure 1: Local IC constraints in the relaxed problem's solution

case, the planner has a strong desire to redistribute resources from the workers in skill groups j and $j+1$ to lower-skilled workers and/or unemployed agents. In the second case, the social planner has a strong desire to redistribute resources from the workers in the higher skill group $j+1$ to the workers in the lower skill group j.

For each pair (α_j, α_{j+1}) in region II, the relaxed problem's solution satisfies both local IC constraints, i.e., the workers in each skill group prefer their own bundle to the one designated for the other group. As can be seen from Figure 1, this constellation mainly occurs if both social weights are close to 1. In this case, the social planner has only a limited desire to redistribute resources between both groups of workers and the average agent in the economy.

For each pair (α_j, α_{j+1}) in region III, the relaxed problem's solution violates the upward IC constraint, i.e., lower-skilled workers consider the bundle $(c_{j+1}^{\alpha R}, y_{j+1}^{\alpha R})$ more attractive than their own bundle. Figure 1 shows that this constellation occurs if both social weights are above the fixed point $\bar{\beta}_j > 1$ and close enough to each other. Hence, the social planner has a strong concern for redistribution from higher-skilled workers to the workers in the skill groups j and $j+1$, but only a limited desire to redistribute resources between the workers in these two skill groups. By Lemma 5, labor supply in both skill groups is upwards distorted at the extensive margin in this case.

Summarizing, Lemma 6 clarifies that the relaxed problem's solution may indeed conflict with downward incentive-compatibility, as one might expect. But it may also conflict with upward incentive compatibility even if the social weight sequence is strictly decreasing, which is true for all weights in $\mathcal{A}^\chi$. In this case, upwards distortions at the intensive margin would be optimal if the social planner only had to account for the IC constraints between skill groups j and $j+1$. The main results in the following subsection provide conditions under which the same is true in the optimal (second-best) allocation, i.e., when the full set of local IC constraints is taken into account.

5.2 Main results

In the following, I characterize the labor supply distortions in the optimal allocation, i.e., the solution to the non-relaxed problem. I start by identifying a set of properties that the optimal allocation satisfies for any decreasing weight sequence in set $\mathcal{A}^{\chi}$.

Proposition 1. *For each $\alpha \in \mathcal{A}^{\chi}$, the optimal tax problem has a unique solution (c^{α}, y^{α}) with $\delta_j \in [\underline{\delta}, \bar{\delta})$ for all $j \in J$. In this solution,*

(i) the consumption level c_0^{α} of the unemployed is strictly positive;

(ii) there is a number $k^{\alpha} \in (0, n)$ such that optimal output is

a) upwards distorted at the extensive margin in skill group j if and only if $j \leq k^{\alpha}$, and

b) downwards distorted or undistorted at the intensive margin in skill group j if $j > k^{\alpha}$;

(iii) optimal output in the highest skill group n is undistorted at the intensive margin and downwards distorted at the extensive margin.

By Proposition 1, the optimal tax problem has a well-defined solution with the following properties for any $\alpha \in \mathcal{A}^{\chi}$: First, the social planner provides a strictly positive transfer to the unemployed agents. Second, upward distortions at the extensive margin can only be optimal in the lowest skill groups with $j \leq k^{\alpha}$.[46] Moreover, upward distortions at the intensive margin can only be optimal in a subset of these low-skill groups. Hence, upward distortions at the extensive margin are a necessary, but not sufficient condition for the optimality of upward distortions at the intensive margin. Third, Proposition 1 qualifies the classical *no distortion at the top* result. At the intensive margin, labor supply by the most productive workers is always *undistorted* as in Mirrlees (1971). At the extensive margin, in contrast, labor supply in the top skill group is always *downwards distorted*.

Apart from these common properties, there are substantial differences between the optimal allocations for alternative social weights in $\mathcal{A}^{\chi}$. In the following, I focus on the social weights in specific subsets of $\mathcal{A}_{\chi}$.

[46] It should be noted that this includes the case $k^{\alpha} < 1$, where labor supply in all skill groups is downwards distorted at the extensive margin (i.e., that $k^{\alpha} < 1$).

Definition 1. *For each $k \in \{2, \ldots, n-1\}$, set $\mathcal{A}_k^U$ contains all welfare weight sequences $\alpha \in \mathcal{A}^\chi$ such that*

(i) $\alpha_{j+1} \geq \beta_j^U(\alpha_j)$ for all $j \in \{1, \ldots, k-1\}$ with at least one strict inequality, and

(ii) $\alpha_{j+1} \geq \beta_j^D(\alpha_j)$ for all $j \in \{k, \ldots, n-1\}$.

The construction of set $\mathcal{A}_k^U$ can be illustrated using Figure 1 above. In this figure, any weight sequence $\alpha \in \mathcal{A}^\chi$ can be depicted as an ascending scatter plot consisting of $n-1$ points, representing the weight-pairs (α_j, α_{j+1}) for all $j \in J_{-n}$. For simplicity, assume that the functions β_j^D and β_j^U were identical for all $j \in J_{-n}$.[47] Then, for any sequence of social weights in $\mathcal{A}_k^U$, the first $k-1$ weight-pairs (α_1, α_2), ..., (α_{k-1}, α_k) are located in region III in the upper right part of Figure 1, while each of the remaining weight-pairs is either located in region III or in the intermediate region II. The social weight sequences hence represents a social planner with a pronounced concern for redistribution from highly skilled workers to the workers in the lowest skill groups and the unemployed, but only a limited concern for redistribution among the workers in the lowest skill groups.

For any $k \in \{2, \ldots, n-1\}$ and any weight sequence in the set $\mathcal{A}_k^U$, the labor supply distortions in the optimal allocation can be characterized as follows.

Proposition 2. *For any $\alpha \in \mathcal{A}_k^U$, optimal output y^α is*

- *upwards distorted at the extensive margin in skill groups $\{1, \ldots, k\}$, and*
- *upwards distorted at the intensive margin in skill groups $\{2, \ldots, k\}$.*

By Proposition 2, the optimal allocation involves upward distortions at both margins for any social weights in the set $\mathcal{A}_k^U$. In particular, labor supply in the k lowest skill groups is upwards distorted at the extensive margin, and labor supply by all workers with skill types $\omega_j \in \{\omega_2, \ldots, \omega_k\}$ is upwards distorted at the intensive margin. For all social weight in $\mathcal{A}_k^U$ with any $k \geq 2$, this optimal allocation can only be decentralized by an income tax with negative marginal taxes and negative participation taxes at low income levels. Proposition 2 hence provides a sufficient condition for the optimality of an *EITC*, expressed in terms of social welfare weights only. In Appendix B.1, I complement this result by two necessary conditions for the optimality of an *EITC*.[48]

It is important to emphasize, however, that these social weights are endogenous objects that depend on the properties of the welfare function (8) and on the joint type

[47] Note that this simplifying assumption is only used to explain the construction of set $\mathcal{A}_k^U$.

[48] In particular, I identify social welfare weights for which the optimal marginal income tax is (a) strictly positive everywhere below the top or (b) zero at all relevant income levels.

distribution K. Hence, the previous result is only relevant if there exist well-behaved welfare functions for which a weight sequence $\bar{\alpha}$ in the set $\mathcal{A}_k^U$ arises endogenously (for some $k \in J$). In the following, I focus on welfare functions that involve a transformation $\Psi : \mathbb{R} \to \mathbb{R}$ and a type-dependent weighting function $\gamma : \Omega \times \Delta \to \mathbb{R}_+$ with standard properties. More precisely, a welfare function (Ψ, γ) is said to be *regular* if (a) Ψ is strictly increasing and weakly concave and (b) γ is weakly decreasing in ω and weakly increasing in δ.

Proposition 3. *There are two numbers $a_U^k > 1$, $m^k \geq k+1$ and two vectors $(\phi_j^k)_{j=1}^n$, $(\delta_j^k)_{j=1}^n$ with $\phi_{j+1}^k \geq \phi_j^k$ for all $j \in J_{-n}$, $\phi_j^k \gtrless 1$ for $j \gtrless m^k$ and $\delta_j^k \in \left(\underline{\delta}, \bar{\delta}\right)$ for all $j \in J$ such that, if*

(a) $\frac{\omega_{j+1}}{\omega_j} < a_U^k$ for all $j \in \{1, \ldots, k-1\}$,

(b) $n \geq m^k$ and

(c) $\sum_{j=1}^n f_j G_j(\delta_j^k)\phi_j^k > 1$,

there exist regular welfare functions for which $\bar{\alpha} \in \mathcal{A}_k^U$.

Proposition 3 provides three conditions that jointly ensure the existence of well-behaved welfare functions for which an *Earned Income Tax Credit* with negative marginal taxes and negative participation taxes is optimal. Although these conditions appear complicated, they can easily be interpreted.

Condition (a) requires the relative distance between each pair of adjacent skill types in Ω to be sufficiently small. It can hence be seen as a technical regularity condition with respect to the skill set Ω.

The remaining two conditions ensure that there is a sufficiently large share of workers with higher skill types than ω_k. By condition (b), the cardinality n of the skill set Ω has to be equal to (or above) some finite threshold $m^k \geq k+1$. Recall that the ratio ω_{j+1}/ω_j is assumed to exceed $1 + \varepsilon$ for some $\varepsilon > 0$. Hence, condition (b) requires the relative difference between the highest skill type ω_n and the lowest skill type ω_1 to be large enough.

By condition (c), the population share of the agents with high skill types (ω_{m^k} or higher) and low fixed cost types has to be sufficiently large. To see this, note that the condition compares a weighted average over the increasing sequence $\phi^k = (\phi_1^k, \phi_2^k, \ldots, \phi_n^k)$ with 1. Each element ϕ_j^k is weighted by the population share of the agents with skill type ω_j and fixed cost types below some threshold δ_j^k (i.e., the agents in skill group j with the largest preference for participating in the labor market). By construction, element ϕ_j^k is smaller than 1 for all skill groups below the threshold m^k, and larger than 1 for all higher skill groups. Hence, condition (c) is certainly satisfied if the population share $\sum_{j=m^k}^{n} f_j G_j(\delta_j^k)$ of highly productive agents is close to 1, and certainly violated if the same population share is close to zero.

For a given joint type distribution K, the threshold m^k is increasing in the level of k, i.e., the number of skill groups for which the optimal marginal tax is negative. Thus, the larger k is, the harder conditions (b) and (c) are to satisfy. As a result, there is a unique critical value $\bar{k} \leq n-1$ such that both conditions are jointly satisfied if and only if k is below $\bar{k}$. Put differently, Proposition 3 implies that the optimal allocation can only involve upwards distortions at the intensive margin for workers with some critical skill type $\omega_{\bar{k}}$ and below.[49]

From an applied perspective, results on the exact location of the critical skill $\omega_{\bar{k}}$ and, correspondingly, on the income range with optimally negative marginal taxes would be of primary interest. In principle, the conditions in Proposition 3 allow to determine $\omega_{\bar{k}}$ precisely, given appropriate information about the primitives of the model, especially the effort cost function and the joint type distribution. Unfortunately, these objects cannot be observed directly and the critical value $\omega_{\bar{k}}$ cannot be determined analytically. In the next sections and in Appendix B, however, I use three strategies to derive additional insights on the optimality of upwards distortions at the intensive margin.

First, I calibrate the model to the US economy in Section 7, matching empirical moments such as the income distribution and labor supply elasticities. For the calibrated model, I am able to determine the critical skill level $\omega_{\bar{k}}$ and the corresponding (phase-in) income range numerically. Second, Lemma 25 in Appendix B.4 provides an upper bound on the critical skill $\omega_{\bar{k}}$, which is mainly expressed in terms of observable quantities such as the joint type distribution and labor supply elasticities. Third, I use this upper bound to show that the potential optimality of negative marginal taxes remains valid if the skill set converges from a discrete set to an interval in Appendix B.5.[50] These additional results hence suggest that the main result of this paper – the potential optimality of negative marginal taxes – is both robust and empirically relevant.

[49]Hence, the endpoint of the *EITC* phase-in range is never located above the optimal income of workers in threshold group $\bar{k}$.

[50]For the third point, I focus on an example with simple functional forms that allows to derive a limit result on the skill range with optimally negative marginal taxes.

6 The tradeoff between intensive efficiency and extensive efficiency

In the following section, I explain the economic mechanism behind Propositions 1 to 3. The optimal pattern of labor supply distortions are driven by, first, the standard trade-off between equity and efficiency and, second, a previously neglected trade-off between labor supply distortions at both margins. The section focuses on an auxiliary problem that helps to isolate the latter trade-off and clarify its implications for the optimal allocation.[51]

In particular, consider the auxiliary problem to maximize efficiency subject to a reduced set of incentive compatibility constraints and to the constraint that some fixed amount of resources is redistributed from the high-skill workers to the unemployed agents and the low-skill workers. More precisely, the planner's problem is to minimize the deadweight loss from labor supply distortions (5) over the set of feasible allocations, subject to the constraint that the exogenous amount $R > 0$ of resources is transferred from the set of workers with skill type ω_3 and higher to the set of unemployed agents and workers with skill types ω_1 and ω_2,

$$\sum_{j=1}^{n} f_j \left[1 - G_j(\delta_j)\right] c_0 + \sum_{j=1}^{2} f_j G_j(\delta_j)(c_j - y_j) = \sum_{j=3}^{n} f_j G_j(\delta_j)(y_j - c_j) = R \ , \quad (21)$$

and to the incentive compatibility constraints between all agents with identical skills (13), between all unemployed agents (14), and between the workers in the lowest two skill groups,

$$c_2 - h(y_2, \omega_2) \ \geq \ c_1 - h(y_1, \omega_2) \ , \quad (22)$$

$$c_1 - h(y_1, \omega_1) \ \geq \ c_2 - h(y_2, \omega_1) \ . \quad (23)$$

I henceforth refer to this program as the problem of efficient redistribution. I denote its solution by (c^E, y^E) and the implied vector of participation thresholds by δ^E. The following lemma identifies the labor supply distortions in this solution.

Lemma 7. *Consider a redistribution amount $R > 0$ such that the solution (c^E, y^E) to the efficient distribution problem exists and is interior.*

(i) Output y^E is upwards distorted at the extensive margin in skill groups 1 and 2.

[51] At the end of this section, I also comment on the differences between this auxiliary problem and the optimal tax problem studied above.

(ii) *There is a number* $a^E > 1$ *such that, if* $\omega_2/\omega_1 \in \left(1, a^E\right)$*, output* y^E *is upwards distorted at the intensive margin in skill group* 2*.*

By Lemma 7, redistributing resources in the most efficient way requires to distort labor supply of low-skill workers upwards at both margins. This result holds whenever, first, the distance between skill groups 1 and 2 is sufficiently small and, second, the problem has an interior solution. The first qualification is related to the assumption of a discrete skill set and will become clear below. The second qualification has to be made because the problem may fail to have a well-behaved solution for high levels of R. In particular, it may be impossible to collect the required amount of resources from the high-skilled workers due to Laffer curve effects.[52] Besides, the solution for high levels of R may involve labor market participation by all low-skill agents, i.e., a boundary solution with extreme upward distortions. As both problems have no relevance for the optimal income tax problem and the trade-off between intensive efficiency and extensive efficiency, I henceforth restrict my attention to cases with a well-behaved solution.

Upward distortions at the extensive margin. I start by explaining why efficient redistribution y^E gives rise to upward distortions at the extensive margin in both low-skill groups, i.e., why the participation threshold δ_j^E exceeds its first-best level $\delta^*(\omega_j)$ for $j \in \{1, 2\}$ (first part of Lemma 7). Assume first that the local IC constraints between the workers in skill groups 1 and 2 are not binding and can hence be ignored. In this case, the efficiency-maximizing allocation (c^E, y^E) does not involve distortions at the intensive margin. Hence, the social planner only faces the problem to minimize the deadweight loss from distortions at the extensive margin.

For each $j \in \{1, 2\}$, the optimal level of the participation threshold δ_j^E is implicitly defined by the first-order condition with respect to c_j,

$$\delta_j^E - \delta^*(\omega_j) = c_j^E - y_j^E - c_0^E = \frac{\lambda_E}{1-\lambda_E} \, \frac{1}{\eta_j(c^E, y^E)} > 0 \, , \tag{24}$$

where λ_E is the Lagrange multiplier associated with the constraint that R resources have to be transferred to the unemployed and the working poor. Equation (24) has two crucial implications.

First, low-skill labor supply is upwards distorted at the extensive margin whenever the redistribution constraint is binding, i.e., the amount R is strictly positive.[53]

[52] The more resources are transferred from high-skill workers to unemployed agents, the more high-skill workers become unemployed. Hence, the level of transfers is bounded from above.

[53] Note that λ_E takes a value in the interval $(0, 1)$ for any $R > 0$ such that (c^E, y^E) is interior.

Put differently, efficient redistribution always involves larger transfers to the low-skilled workers than to the unemployed. To provide the economic intuition behind this result, consider an initial allocation in which labor supply in both low-skill groups is undistorted at both margins. This requires that, first, the output levels y_1 and y_2 satisfy $h_y(y_1, \omega_1) = h_y(y_2, \omega_2) = 1$ and, second, identical transfers are provided to the low-skill workers and the unemployed agents, $c_1 - y_1 = c_2 - y_2 = c_0 > 0$. Feasibility requires that these transfers are paid by the high-skill workers, i.e., $y_j - c_j > 0$ for all $j \geq 3$. Hence, labor supply in the high-skill groups must be downwards distorted at the extensive margin.

Assume now that the planner reduces the consumption level c_0 of the unemployed and increases the consumption levels c_1 and c_2 of the working poor in a budget-balancing way, holding R constant. This has two effects on labor supply. First, some previously unemployed agents in both low-skill groups start working, creating an upwards distortion at the extensive margin. Initially, this only leads to a negligible (second-order) increase in the deadweight loss, because labor supply in these groups was undistorted before. Second, some previously unemployed agents in all high-skill groups start working due to the reduction in c_0. This response leads to a first-order reduction in the deadweight loss, because labor supply was downwards distorted at the extensive margin before and is less so now. Hence, providing larger transfers to the low-skilled workers than to the unemployed increases extensive efficiency, although it leads to upward distortions at the extensive margin.

Second, the first-order conditions with respect to c_1 and c_2 imply that the workers in both low-skill groups receive different transfers. It is worth noting that, if the local IC constraints between both low-skill groups are ignored, the problem of efficient redistribution is structurally identical to the Ramsey problem of optimal commodity taxation. Accordingly, equation (24) represents an *inverse elasticity rule*: The transfer to skill group $j \in \{1, 2\}$ has to be proportional to the inverse of the semi-elasticity η_j of participation.[54] Recall that the relative sizes of participation responses are pinned down by Condition 1: η_1 exceeds η_2 in every implementable allocation (see Lemma 2). For any $R > 0$, the efficient-maximizing allocation must hence involve strictly higher transfers to the higher-skilled workers than to the lower-skilled workers, $c_2^E - y_2^E > c_1^E - y_1^E$.

Upward distortions at the intensive margin. By the previous paragraph, the solution to the problem of efficient redistribution involves higher transfers to the workers in skill group 2 than to the less skilled workers in group 1. This gives rise to

[54] Note that the classical *inverse elasticity rule* is expressed in terms of standard elasticities instead of semi-elasticities.

the question whether the allocation defined by equation (24) and the redistribution constraint (21) violates the upward IC constraint.[55] As Lemma 7 indicates, the answer to this question is positive whenever the skill set is sufficiently "dense", i.e., the distance between skill levels ω_1 and ω_2 is sufficiently small.

Note that the formal derivation of this crucial result involves a non-standard complication. In particular, the violation of the upward IC constraint cannot be verified directly for specific skill distances, as I have not imposed any functional form assumptions on the effort cost function h and the joint type distribution K. The formal proof resolves this problem by studying how the participation threshold δ_2^E is affected by variations in the skill level ω_2. In particular, I first investigate the optimal relation between δ_1^E and δ_2^E for the limit case where ω_2 equals ω_1. Second, I show that the allocation defined by (24) violates the upward IC constraint after a marginal increase in ω_2 whenever Condition 1 is satisfied.

The previous arguments have clarified that the upward IC constraint is binding in (c^E, y^E) if the distance between ω_1 and ω_2 is small enough. Assume that this condition is met. In this case, the social planner cannot set the transfers to both groups of low-skill workers according to the *inverse elasticity rule* (24) and avoid distortions at the intensive margin at the same time. Specifically, to satisfy the *inverse elasticity rule*, he has to relax the upward IC constraint by distorting labor supply y_2 upwards at the intensive margin. Put differently, the planner can only increase extensive efficiency if he reduces intensive efficiency and vice versa. This trade-off constitutes a crucial difference between the problem of efficient redistribution studied here and the standard Ramsey problem.

To minimize the overall deadweight loss (5), the planner has to implement the allocation that equates the marginal deadweight losses from distortions at both margins, representing the optimal compromise between intensive efficiency and extensive efficiency. Formally, the efficiency-maximizing allocation has to satisfy

$$\begin{aligned}\frac{f_2 G_2(\delta_2^E)\left[h_y(y_2^E,\omega_2)-1\right]}{h_y(y_2^E,\omega_1)-h_y(y_2^E,\omega_2)} &= \Lambda\left\{\eta_1\left[\delta_1^E-\delta^*(\omega_1)\right]-\eta_2\left[\delta_2^E-y_2^E+h(y_2^E,\omega_2)\right]\right\} \\ &> 0\ , \end{aligned} \tag{25}$$

where $\Lambda := \left[f_1 f_2 G_1(\delta_1^E) G_2(\delta_2^E)\right] / \left[f_1 G_1(\delta_1^E) + f_2 G_2(\delta_2^E)\right]$.

For the interpretation of this condition, consider a marginal increase in y_2, which relaxes the upward IC constraint and hence allows to raise the difference between the transfers to the workers in skill groups 1 and 2. The left-hand side of equation (25) captures the induced increase in the *intensive* deadweight loss. In particular,

[55] By the previous arguments, the downward IC constraint is trivially satisfied for any $R \geq 0$.

the term in the numerator states the difference between the marginal rate of substitution $h_y(y_2, \omega_2)$ and the marginal rate of transformation 1, while the term in the denominator quantifies the extent to which the upward IC is relaxed.

The right-hand side of (25) captures the reduction in the *extensive* deadweight loss that results from raising the difference between both transfers. In particular, the term in brackets evaluates how much the allocation (c^E, y^E) deviates from the inverse elasticity rule (24). The larger this term is, the more beneficial it is to distort y_2 upwards in order to increase the difference between both transfers.[56]

Summing up, the solution to the auxiliary problem of efficient redistribution involves upward distortions in labor supply at both margins. Note that these insights extend to a more general version of the efficient redistribution problem in which the planner wants to redistribute resources to the unemployed and the workers with the lowest $k \in (2, n)$ skill types (from all higher-skilled workers), and takes into account the local IC constraints between all workers with skill types ω_1 to ω_k. In this case, the efficiency-maximizing allocation involves upward distortions at the extensive margin in the skill groups 1 to k, and upward distortions at the intensive margin in the skill groups 2 to k. As shown in Subsection 3.3, this allocation can be decentralized through an *EITC*-type income tax schedule with negative participation taxes and negative marginal taxes for low-skill workers.

Finally, it should be noted that the tradeoff between intensive and extensive efficiency also provides the intuition for the potential optimality of an *EITC* in the optimal tax problem (see Propositions 1 to 3). Of course, this tradeoff does not provide a complete explanation for my results. In particular, the optimal tax problem differs in two crucial aspects from the auxiliary problem studied above: First, the social planner has concerns for redistribution among the poor, i.e., between the working poor and the unemployed. Second, the planner has to take into account all local incentive-compatibility constraints, instead of only those between low-skill workers. Intuitively, both aspects tend to increase the optimal levels of marginal taxes and participation taxes. Propositions 2 and 3 and their formal proofs in Appendix A show, however, that an *EITC* with negative marginal taxes and negative participation taxes remains optimal for all *regular* welfare functions that give rise to welfare sequences in any of the sets $\mathcal{A}_2^U$, $\mathcal{A}_3^U$, etc.[57]

[56]Note that equation (25) is also satisfied if the distance between both skill levels ω_1 and ω_2 is above the threshold a_E. In this case, allocation (c^E, y^E) satisfies the inverse elasticity rule and involves no distortion at the intensive margin. Thus, both sides of the equation equal zero.

[57]For the first aspect, Lemmas 5 and 6 show that upwards distortions at both margins remain optimal if and only if the concerns for redistribution among the poor are sufficiently weak. For the second aspect, Appendix B.10 explains why optimal labor supply in skill groups 2 to k remains upwards distorted for all $\alpha \in \mathcal{A}_k^U$, even if the entire set of IC constraints is taken into account.

7 Numerical simulations

The theoretical analysis above has shown that an *EITC* with negative marginal taxes and negative participation taxes at the bottom of the income distribution can be optimal given *regular* welfare functions. Given its generality, this analysis cannot provide clear insight on quantitative aspects of this result, such as the income range on which an *EITC* should apply and the optimal levels of negative marginal and participation taxes. To make progress on these quantitative questions, I calibrate the model to match a set of empirical moments for the US economy. I use this calibrated model to numerically simulate the optimal income tax for specific sequences of welfare weights. I find that an optimal *EITC* can be quantitatively large in terms of the income range as well as the levels of (negative) marginal and participation tax rates.

7.1 Calibration

To calibrate the model to the US economy, I target empirical estimates of the labor supply elasticities at both margins and the income distribution among workers in the US. This requires to impose further assumptions on the individual preferences and the joint type distribution of skills and fixed costs of working. I focus on childless singles in the US in order to remain consistent with the theoretical model studied above, which does not account for labor supply decisions within families. To simplify comparisons with the previous literature, I closely follow the calibration by Jacquet et al. (2013).

First, a large literature estimates the elasticity of labor income with respect to the retention rate $1 - T'(y)$ among workers. According to the survey by Saez et al. (2012), the best available estimates are in the range between .12 and .4. To match these numbers, I assume that the effort cost function is given by

$$h(y^i, \omega^i) = \frac{\sigma}{1+\sigma} \left(\frac{y^i}{\omega^i} \right)^{1+1/\sigma} . \tag{26}$$

For this functional form, the elasticity of income with respect to the retention rate is equal to parameter σ for all individuals. I set σ equal to .3, slightly above the center of the range of empirical estimates.[58]

Second, the empirical literature consistently finds that participation elasticities

[58]Recall that the quasi-linearity of the utility function (1) assumes away income effects in labor supply. Because empirical studies tend to find small and often insignificant income effects, this seems to be an acceptable simplification (see also Jacquet et al. 2013).

are decreasing over the skill distribution. There is less consensus about the levels of these elasticities, with estimates ranging from .06 to 1 (e.g., Juhn et al. 1991, 2002, Eissa & Hoynes (2004) and Meghir & Phillips 2010). Recall that the participation elasticity in each skill group depends on the skill-specific distribution of fixed cost. Unfortunately, there is no evidence on the shape of these fixed cost distributions to the best of my knowledge. Given the lack of better alternatives, I calibrate the conditional fixed cost distributions exactly as in Jacquet et al. (2013).

In the first step, I specify the pattern of skill-specific participation elasticities and employment shares to target. In particular, I assume that the participation elasticity $\pi_j = A_j(\delta_j)(c_j - c_0)$ and the employment rate $L_j = G_j(\delta_j)$ in skill group j are given by

$$\pi_j = .5 - .1 \left(\frac{\omega_j - \omega_1}{\omega_n - \omega_1} \right)^{1/3} \quad \text{and} \quad L_j = .7 + .1 \left(\frac{\omega_j - \omega_1}{\omega_n - \omega_1} \right)^{1/3} , \tag{27}$$

respectively (under the current US tax system). Hence, skill-specific participation elasticities decrease from .5 in the lowest skill group to .4 in the highest skill group. In contrast, skill-specific employment rates increase from .7 in the lowest skill group to .8 in the highest skill group.[59]

In the second step, I assume that fixed costs are distributed according to a logistic distribution of the form

$$G_j(\delta) = \frac{\exp\left(-\psi_j + \rho_j \delta\right)}{1 + \exp\left(-\psi_j + \rho_j \delta\right)} . \tag{28}$$

in each skill group j. For each $j \in J$, the parameters ψ_j and ρ_j are chosen to match the levels of the participation elasticity π_j and the employment share L_j specified by (27). Note that the functional form of (28) ensures that the employment share is between 0 and 1 and that labor supply responds at the extensive margin in each skill group for any admissible tax function T.

I calibrate the unconditional skill distribution to match the observed income distribution in the US economy. I estimate the latter distribution based on labor income data for childless singles at ages 25 to 60 in the March 2016 CPS.[60] Using the OECD tax database, I approximate the US income tax in 2015 for childless singles by a linear tax function with marginal tax rate 29.3% (OECD 2017).[61] Based on

[59] In the calibrated model, the unconditional share of non-workers among childless singles is given by 23.4% under the current US tax system.

[60] In particular, I compute for each worker his average income for each week in employment according to the CPS data. To calculate an agent's skill, I then multiply the weekly income by 52 to get individually optimal incomes conditional on working the entire year.

[61] The tax approximation is similar to the one in Jacquet et al. (2013), again. It takes into

this approximation, I can use the first-order condition of the individual optimization program to back out the skill type of each CPS respondent with strictly positive labor income.

I consider a discrete skill set with $n = 96$ skill types, where the relative distance between each pair of adjacent skill types is equal to $\omega_{j+1}/\omega_j = 1.05$. Compared to most previous papers, this can be considered as a relatively fine or "dense" skill set. The lowest and highest skill types receives wagesof $\omega_1 = \$129$ and $\omega_n = \$13,300$ per unit of work, corresponding to yearly incomes of \$500 and \$206,942, respectively.[62] To obtain a smooth distribution, I estimate the share of workers in each skill group j with a kernel density approximation of the distribution of computed skill types. This procedure gives the conditional skill distribution among employed workers under the 2015 US tax regime. In the last step, I use the pattern of skill-specific employment rates imposed by (27) to compute the unconditional skill distribution among all childless singles (workers and unemployed).

Finally, I study the optimal income tax for specific redistributive preferences that allow to illustrate the theoretical results of this paper and to assess their quantitative relevance.[63] More precisely, I construct two sequences α^A and α^B of welfare weights that (i) are monotonically decreasing over the income distribution, (ii) are elements of the sets $\mathcal{A}^U_{k_A}$ and $\mathcal{A}^U_{k_B}$ according to Definition 1 (where k_A and k_B are two numbers below n), and (iii) satisfy the conditions in Proposition 3. Consequently, I already know that (ii) the optimal tax involves in both cases some kind of *EITC* from Proposition 2, and that (iii) both sequences can arise endogenously for *regular* welfare functions with standard properties from Proposition 3. As Figure 3 in Appendix B.2 shows, both sequences give rise to only weak (or no) concerns for redistribution among the poor, i.e., from low-income earners to even-lower-income earners and to the unemployed.[64] It is important to emphasize that Saez (2002) and Jacquet et al. (2013) focus on social objectives that imply stronger concerns for redistribution among the poor, in contrast.[65]

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account central and (average) state income taxes as well as employee social security contributions (OECD 2017, see also http://www.oecd.org/tax/tax-policy/tax-database.htm). Note that this linear tax underestimates the marginal tax wedge for high-income earners to some extent.

[62]In the March 2016 CPS, the computed skill of about .1% single workers is below ω_1, while the computed skill of about 1.3% single workers is above ω_n.

[63]For this purpose, I set the exogenous revenue requirement to 0 as imposed by (3). Hence, I assume that the government uses income taxation for redistributive purposes only.

[64]The differences between α^A and α^B are explained in Subsection 7.2 below.

[65]Jacquet et al. (2013) consider Utilitarian (Benthamite) social preferences with a constant curvature in individual utilities, which translates into equally strong concerns for redistribution over the entire income distribution. Saez (2002) considers marginal welfare weights that are decreasing and strictly convex, so that redistributive concerns among low-income earners are stronger than among medium- or high-income earners.

7.2 Simulation results

In the following, I provide the simulation results for the social weight sequences α^A and α^B. These sequences are constructed to provide insights on two quantitative aspects of my theoretical results. First, which *levels* of negative marginal taxes and negative participation taxes can be optimal, given well-behaved social preferences? Second, on which *income ranges* can the optimal marginal taxes be negative, i.e., which phase-in ranges can be optimal?

To answer the first question, I consider weight sequence α^A, which associates identical welfare weights to all workers with incomes in the phase-in range of the 2015 *EITC* for childless workers, i.e., with incomes below \$6.580.[66] Sequence α^A hence assumes away concerns for redistribution among the working poor. By Proposition 2, this ensures the optimality of negative marginal taxes at low incomes.

To answer the second question, I exploit that Proposition 3 provides sufficient conditions for the optimality of negative marginal taxes under *regular* welfare functions with standard properties. I construct weight sequence α^B so to maximize the phase-in range, i.e., the income range with negative marginal taxes. The resulting sequence implies very small concerns for redistribution among the low-skill workers with incomes up to the endpoint of the phase-in range.[67]

Figure 2 on the next page illustrates the simulation results for weight sequences α^A and α^B by depicting the optimal participation taxes $T_A^P(y) = T_A(y) - T_A(0)$ and $T_B^P(y) = T_B(y) - T_B(0)$ for annual incomes below \$50,000. It proves helpful to compare the simulation results with the properties of the US income tax system as a benchmark (red dotted line in Figure 2). In 2015, childless singles were eligible for the *EITC* if their earned income was below \$14,820. The marginal income tax for this group was -7.65% for incomes below \$6.580 (phase-in range) and $+7.65\%$ for incomes between \$8.240 and \$14.820 (phase-out range). Workers with incomes between \$6.580 and \$8.240 received the maximum tax credit of \$503.[68]

First, the optimal income tax for weight sequence α^A involves an *EITC* with

[66] In particular, I set weight $\alpha_j^A = 1.05$ for all skill groups $j \leq 41$ (under laissez-faire, the income of workers in skill group 41 is equal to \$7.014). The weights of all higher skill groups are set to ensure that α^A is an element of $\mathcal{A}_{41}^U$ (see Definition 1). For further details, see Appendix B.2.

[67] More precisely, Proposition 3 implicitly defines the highest skill group $\bar{k} \in J$ for which, given a *regular* welfare function, labor supply may be upwards distorted at the intensive margin. Sequence α^B is hence constructed to be an element of the set $\mathcal{A}_{k_B}^U$ with $k_B = \bar{k}$, ensuring negative marginal taxes for the workers in skill groups 2 to $\bar{k}$. The exact construction of sequences α^A and α^B is explained more formally in Appendix B.2.

[68] Note that, in the phase-in region, the *EITC* rate exactly offsets the marginal social security contributions (employee share). *EITC* rates and payments are larger for single parents and married couples with children. Maag et al. (2012) show that, for these groups, effective marginal taxes are around -20% in some US states and strictly positive in other states if transfer programs such as TANF and SNAP are taken into account.

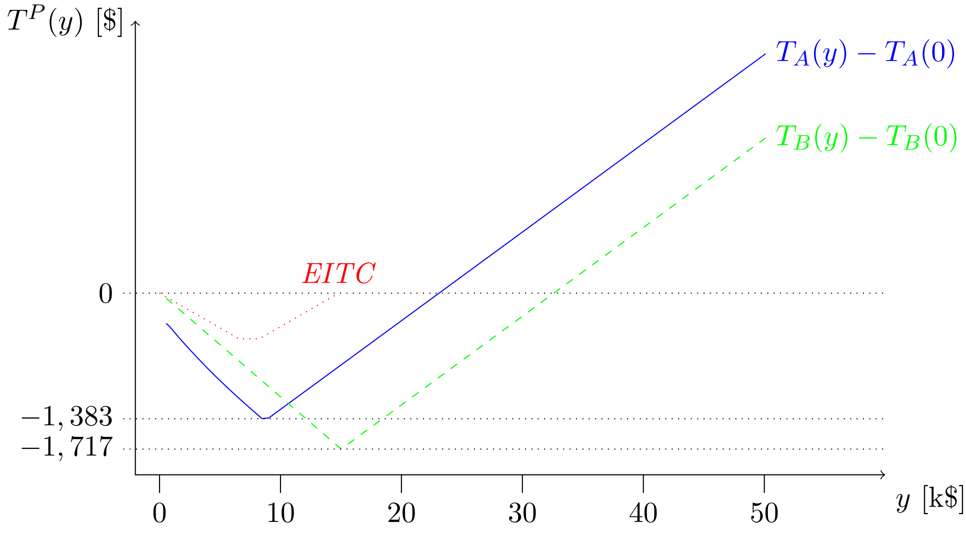

Figure 2: Optimal participation tax functions T_A^P and T_B^P.

negative marginal taxes for all income levels up to $y_{44}^A = \$8,485$, and negative participation taxes for all incomes up to $y_{59}^A = \$21,970$ (see solid blue line in Figure 2). In the calibrated model, 14.3% of all childless singles benefit from the optimal *EITC*, and 18.1% of these recipients have incomes in the phase-in range. The share of non-working agents is reduced to 12.9%.

For the optimal income tax function, the maximum tax credit is given by $1,383 and reached at income level $8,485. For comparison, the optimal unemployment benefit is given by $c_0^A = \$3,019$. The ratio $T_A^P(y)/y$ of optimal participation taxes to pre-tax incomes, which is sometimes referred to as the participation tax rate, is around −60% for very low incomes such as $y_1^A = \$555$.[69] The ratio subsequently diminishes to levels around −16% at the phase-in endpoint. As in any optimal tax model with a discrete skill set, marginal taxes can be measured in two alternative ways. On the one hand, the average marginal tax in the phase-in range is equal to −13,2%. Between income levels $y_{15}^A = \$1,365$ and $y_{16}^A = \$1,455$, it even decreases to −16.2%. On the other hand, the implicit marginal taxes are between −4.3% and −2% in the middle of the phase-in range, and between −1% and 0 close to both ends of the phase-in range.[70] In the phase-out range, the average marginal tax is given by 9.5%, while implicit marginal taxes are equal to zero. Hence, labor supply in skill groups $\{2, \dots, 44\}$ is upwards distorted at the extensive margin (substantially) and at the intensive margin (somewhat less).[71]

[69]As visible from Figure 2, these large negative participation taxes suggest that the optimal income tax may fall discontinuously at zero, in line with the results of Jacquet et al. (2013).

[70]The average marginal tax between incomes y_k and y_j is computed as $[T(y_k) - T(y_j)]/(y_k - y_j)$, while the implicit marginal tax at income y_j is given by $1 - h_y(y_j, \omega_j)$.

[71]In the optimal allocation, all local upward IC constraints between skill groups 1 and 44 are

Second, the optimal income tax for weight sequence α^B involves negative marginal taxes over a much larger income range, with the phase-in endpoint given by $y_{53}^B = \$15,016$ (see dashed green line in Figure 2).[72] The optimal participation taxes are even negative up to the income level $y_{65}^B = \$32,144$. In the calibrated model, the recipients of the optimal *EITC* account for about 27.7% of all childless singles, among whom 26% have incomes in the phase-in region. The share of unemployed agents falls to 12,4%.

Regarding the details of the optimal tax function, the maximum tax credit is given by \$1.717, while the optimal unemployment benefit c_0^B is equal to \$2,281. The participation tax rate is between −10% and −11.5% at all income levels in the phase-in range. The average marginal tax rate in the phase-in range is equal to −11.5%, while the implicit marginal tax is negative but close to zero at any income level in this range. Finally, the average marginal tax in the phase-out range is given by 9,7%, while implicit marginal taxes are zero at all income levels above y_{53}^B.[73] The optimal marginal and participation tax rates are hence smaller in magnitudes for weight sequence α_B than for sequence α_A, while the phase-in range, the *EITC* range and the maximum tax credit are larger.

Summing up, the numerical simulations show that the effects of the mechanism studied in this paper - the tradeoff between intensive efficiency and extensive efficiency - are not only qualitatively, but also quantitatively important. When the concerns for redistribution among the poor are weak (as implied by the weight sequences α^A and α^B), the optimal *EITC* may cover a larger income range and feature a larger maximum credit than the current US scheme for childless workers. Moreover, optimal marginal taxes and optimal participation taxes are more negative (larger in absolute terms) than under the *EITC* for childless workers in the US.[74]

Based on these results, two further conclusions can be drawn. First, the reported results imply that one could find another monotonically decreasing sequence of welfare weights for which the optimal income tax is a close approximation of the current *EITC* for childless singles in the US. Put differently, if one would seek to back out the implicit welfare weights of the US government in an inverse optimum exercise

binding, while all local IC constraints between skill groups 44 and 96 are non-binding (the associated Lagrange parameters are zero).

[72] Hence, the highest skill group for which the optimal allocation can involve upward distortions at the intensive margin is given by skill group $\bar{k} = 53$ (given the joint type distribution in the calibrated model). Put differently, I find that *regular* welfare functions cannot give rise to weight sequences in set $\mathcal{A}_k^U$ for any $k > 53$.

[73] Again, all local upwards IC constraints between skill groups 1 and 53 are binding in the optimal allocation, while all local IC constraints between skill groups 53 and 96 are non-binding.

[74] Unreported robustness checks show that these general insights do not depend on the details of the calibration.

as in Jacobs et al. (2017) or Lockwood (2017), the estimated welfare weights would have arguably reasonable properties. Second, it even seems possible to rationalize the recent proposals to strongly expand the *EITC* for childless workers based on a well-behaved welfare function with standard properties, and without reference to behavioral anomalies or paternalistic arguments (for example, see Executive Office 2014 and House Budget Committee 2014).

8 Conclusion

The paper has studied optimal income taxation in an empirically plausible model with labor supply responses at the intensive margin and at the extensive margin. Using a novel modeling strategy, it is the first paper to provide sufficient conditions for the optimality of an *Earned Income Tax Credit* with negative marginal taxes and negative participation taxes at low income levels in such a model. In particular, the optimal income tax is given by an *EITC* if society has strong concerns for redistribution from the rich to the poor, but only weak concerns for redistribution from the poor to the very poor. As shown above, this result is driven by a trade-off between labor supply distortions at the intensive margin and at the extensive margin, which has not been discussed in the previous literature.

Importantly, the paper has shown that an *EITC* can be optimal although society considers the unemployed more deserving than the working poor, and the working poor more deserving than medium-income and high-income earners. It has repeatedly been argued that society might, in contrast, consider the working poor more deserving than the unemployed (for example, see the arguments in Beaudry et al. 2009 and Saez & Stantcheva 2016). As a result of the trade-off between distortions at both margins, redistributive preferences of this kind would not only make a stronger case for negative participation taxes, but also for negative marginal taxes. Hence, the results of this paper would even be reinforced.

Finally, while the paper has confirmed the conjecture of Saez (2002) that an *EITC* can be optimal when labor supply responds at the intensive and extensive margins, it has abstracted from several aspects that could further increase the desirability of such in-work benefit schemes. For example, an *EITC* might have additional benefits if there is learning on the job (Best & Kleven 2013) or if the agents fail to maximize their own well-being due to a presence bias (Lockwood 2017). For future research, it seems worthwhile to investigate the interaction of these non-standard aspects with the mechanism studied in this paper.

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Appendix

A Proofs

Proof of Lemma 1

Proof. Using equation (1), incentive compatibility requires that, for all pairs of (ω, δ) and (ω', δ') in $\Omega \times \Delta$,

$$c(\omega, \delta) - h\left[y(\omega, \delta), \omega\right] - 1_{y(\omega,\delta)>0}\ \delta \geq c(\omega', \delta') - h\left[y(\omega', \delta'), \omega\right] - 1_{y(\omega',\delta')>0}\ \delta\ .$$

To satisfy these constraints, first, all pairs of agents with types (ω_j, δ) and (ω_j, δ') who provide strictly positive output must receive the same gross (of fixed costs) utility $c(\omega_j, \delta) - h\left[y(\omega_j, \delta), \omega_j\right] = z_j$. Second, all agents with types (ω, δ) and (ω', δ') who provide zero output must receive the same consumption $c(\omega, \delta) = c(\omega', \delta') = c_0$. Third, an allocation can only be incentive-compatible if each agent with skill type ω_j and fixed cost type below (above) $\delta_j = z_j - c_0$ provides positive output (zero output).

Second-best Pareto efficiency requires, moreover, that all agents with skill type ω_j and fixed cost type below δ_j receive the same bundle (c_j, y_j). By the strict convexity of effort cost function h, there is a unique bundle (c_j, y_j) that minimizes the net transfer $c - y$, subject to $c - h(y, \omega_j) = z_j$ and to the IC constraints along the skill dimension, $z_k \geq c - h(y, \omega_k)$, for all $k \neq j$. Assume there is an initial allocation in which some agent with type (ω_j, δ) receives bundle $(c', y') \neq (c_j, y_j)$ with $y' > 0$. Changing his allocation to (c_j, y_j) allows to save resources without changing his utility level, and to redistribute these resources lump-sum to all agents in the economy. Hence, the initial allocation with $(c', y') \neq (c_j, y_j)$ was not Pareto efficient. □

Proof of Lemma 2

Proof. In every implementable allocation, the downward IC constraint between the workers with skill types ω_j and ω_{j+1} is satisfied, i.e.,

$$\begin{aligned} & c_{j+1} - h\left(y_{j+1}, \omega_{j+1}\right) && \geq c_j - h\left(y_j, \omega_{j+1}\right) \\ \Leftrightarrow \quad & \delta_{j+1} - \delta_j && \geq h(y_j, \omega_j) - h(y_j, \omega_{j+1})\ . \end{aligned} \tag{29}$$

By $h_\omega(y, \omega) < 0$, this implies that $\delta_{j+1} > \delta_j$. Parts (i) and (ii) of Condition 1 ensure that $\eta_j(c, y) = \frac{g_j(\delta_j)}{G_j(\delta_j)} > \frac{g_j(\delta_{j+1})}{G_j(\delta_{j+1})} \geq \frac{g_{j+1}(\delta_{j+1})}{G_{j+1}(\delta_{j+1})} = \eta_{j+1}(c, y)$. □

Proof of Lemma 3

Proof. Fix some implementable allocation (c, y) with skill-specific participation thresholds $\delta_j = c_j - h(y_j, \omega_j) - c_0$ and $\delta_{j-1} = c_{j+1} - h(y_{j+1}, \omega_{j+1}) - c_0$, respectively. Assume now that the social planner provides an additional, uniform transfer $c' \geq 0$ to the workers in both skill groups. Then, the ratio of relative participation responses is given by

$$\hat{\eta}_{j,j+1}(\delta_j, \delta_{j+1}, c') = \frac{g_j(\delta_j + c')}{G_j(\delta_j + c')} \frac{G_{j+1}(\delta_{j+1} + c')}{g_{j+1}(\delta_{j+1} + c')} \; .$$

The partial derivative of $\hat{\eta}_{j,j+1}(\delta_j, \delta_{j+1}, c')$ with respect to c' is given by

$$\begin{aligned} \frac{\partial \hat{\eta}_{j,j+1}}{\partial c'} &= \left[\frac{\partial \eta_j}{\partial \delta_j} \frac{1}{\eta_j} - \frac{\partial \eta_{j+1}}{\partial \delta_{j+1} \frac{1}{\eta_{j+1}}} \right] \frac{\eta_j}{\eta_{j+1}} \\ &= \left[\left(a_j(\delta_j + c') - A_j(\delta_j + c') \right) - \left(a_{j+1}(\delta_{j+1} + c') - A_{j+1}(\delta_{j+1} + c') \right) \right] \hat{\eta}_{j,j+1} \; . \end{aligned}$$

Setting $c' = 0$, the elasticity of $\hat{\eta}_{j,j+1}(\delta_j, \delta_{j+1}, c')$ with respect to c' follows as

$$\varepsilon_{\hat{\eta},c'} = \frac{\partial \hat{\eta}_{j,j+1}}{\partial c'} \frac{1}{\hat{\eta}_{j,j+1}} = a_j(\delta_j) - a_{j+1}(\delta_{j+1}) - A_j(\delta_j) + A_{j+1}(\delta_{j+1}) \; .$$

Under Condition 2, the last expression is bounded from below by $-\left[A_j(\delta_j) - A_{j+1}(\delta_{j+1})\right] = \eta_{j+1} - \eta_j < 0$ and from above by $A_j(\delta_j) - A_{j+1}(\delta_{j+1}) = \eta_j - \eta_{j+1} > 0$. □

Proof of Lemma 4

Proof. For part i, consider some implementable allocation (c, y) and some working agent i with $\omega^i = \omega_j$ and $\delta^i < \delta_j$. Assume as usual that the tax function T is linear at income level $y^*_T(\omega_j)$, which is defined by $h_y(y^*_T(\omega_j), \omega_j) = 1 - T'\left(y^*_T(\omega_j)\right)$. For this agent, the elasticity of income $y_T(\omega^i, \delta^i)$ with respect to the retention rate $r = 1 - T'\left(y^*_T(\omega_j)\right)$ is given by

$$\varepsilon_{y,1-T'} = \frac{\partial y^*_T(\omega_j)}{\partial r} \frac{r}{y^*_T(\omega_j)} = \frac{1}{h_{yy}(y^*_T(\omega_j), \omega_j)} \frac{h_y(y^*_T(\omega_j), \omega_j)}{y^*_T(\omega_j)} \; ,$$

which is weakly below some $\mu_1 \in (0, \infty)$ for all $\omega_j \in \Omega$ and $r > 0$ by Condition 3 (i).

For part ii, consider the same allocation (c, y) and agent i. The elasticity of income with respect to his skill level ω_j is given by

$$\varepsilon_{y,\omega} = \frac{\partial y^*_T(\omega_j)}{\partial \omega} \frac{\omega}{y^*_T(\omega_j)} = \frac{h_{y\omega}(y^*_T(\omega_j), \omega_j)}{h_{yy}(y^*_T(\omega_j), \omega_j)} \frac{\omega_j}{y^*_T(\omega_j)} \; ,$$

which is weakly above some $\mu_2 \in (0, \infty)$ for all $\omega_j \in \Omega$ and $r > 0$ by Condition 3 (ii). □

Proof of Lemma 5

The proof of Lemma 5 consists of three steps. I start with two preliminary Lemmas.

Lemma 8. *Consider the relaxed problem formally defined in section 5.1. For any* $\alpha \in \mathcal{A}$ *such that an solution* $(c^{\alpha R}, y^{\alpha R})$ *with* $\delta_j^{\alpha R} \in (\underline{\delta}, \bar{\delta})$ *for all* $j \in J$ *exists, it satisfies conditions* (15), (16) *and* (17).

Proof. Assume that the relaxed problem has a solution for social weights α. First, $(c^{\alpha R}, y^{\alpha R})$ must be Pareto-efficient in the set of allocations that are feasible and satisfy the IC constraints (13) and (14). Second, the statements in 1 must be true for $(c^{\alpha R}, y^{\alpha R})$ because its proof only uses the IC constraints (13) and (14). Consequently, the Lagrangian of the relaxed problem can be written as

$$\begin{aligned} \mathcal{L} &= \sum_{j=1}^{n} f_j \Big(\int_{\underline{\delta}}^{\delta_j} g_j(\delta)\gamma(\omega,\delta)\Psi\left[c_j - h(y_j,\omega_j) - \delta\right] d\delta + \int_{\delta_j}^{\bar{\delta}} g_j(\delta)\gamma(\omega,\delta)\Psi(c_0)\ d\delta \Big) \\ &\quad +\lambda \left[\sum_{j=1}^{n} f_j G_j(\delta_j)(y_j - c_j + c_0) - c_0 \right], \end{aligned}$$

where $\delta_j = c_j - h(y_j, \omega_j) - c_0$ and λ is the Lagrange parameter associated with the feasibility condition. Replacing the average weights $\bar{\alpha}_j$ and $\bar{\alpha}_0$ as defined in (9) and (10) by the exogenous numbers α_j and α_0, the first-order conditions of this problem are given by

$$\begin{aligned} \mathcal{L}_{c_j} &= f_j \left[G_j(\delta_j^{\alpha R})\,(\alpha_j - \lambda) + \lambda g_j(\delta_j^{\alpha R})(y_j^{\alpha R} - c_j^{\alpha R} + c_0^{\alpha R}) \right] && \overset{!}{=} 0 \\ \mathcal{L}_{y_j} &= f_j \left[-h_y(y_j^{\alpha R}, \omega_j)\Big(G_j(\delta_j^{\alpha R})\alpha_j + \lambda g_j(\delta_j^{\alpha R})(y_j^{\alpha R} - c_j^{\alpha R} + c_0^{\alpha R}) \Big) + \lambda G_j(\delta_j^{\alpha R}) \right] && \overset{!}{=} 0 \\ \mathcal{L}_{c_0} &= \sum_{j=1}^{n} f_j \left[\Big(1 - G_j(\delta_j^{\alpha R})\Big)\alpha_0 + \lambda G_j(\delta_j^{\alpha R}) - \lambda g_j(\delta_j^{\alpha R})(y_j^{\alpha R} - c_j^{\alpha R} + c_0^{\alpha R}) - 1 \right] && \overset{!}{=} 0 \end{aligned}$$

Equation (15) follows from combining the first-order conditions with respect to c_j and y_j. Combining the FOCs with respect to c_j for all $j \in J$ and c_0 gives

$$\lambda = \sum_{j=1}^{n} f_j \left[G_j(\delta_j^{\alpha R})\alpha_j + \Big(1 - G_j(\delta_j^{\alpha R})\Big)\alpha_0 \right] = \alpha_M = 1,$$

where the average weight α_M is normalized to 1 for all $\alpha \in \mathcal{A}$. Using $\lambda = 1$ and $c_j = \delta_j + h(y_j, \omega_j) + c_0$, the FOC with respect to c_j can be rewritten as

$$\delta_j^{\alpha R} - y_j^{\alpha R} + h(y_j^{\alpha R}, \omega_j) = \frac{G_j(\delta_j^{\alpha R})}{g_j(\delta_j^{\alpha R})}(\alpha_j - 1). \tag{30}$$

As $A_j(\delta) = \frac{g_j(\delta)}{G_j(\delta)}$ and $\delta^*(\omega_j) = \max_{y>0}\{y - h(y, \omega_j)\} = y_j^{\alpha R} - h(y_j^{\alpha R}, \omega_j)$, this is identical to equation (16). Finally, equation (17) follows from substituting $-\delta_j^{\alpha R} - h(y_j^{\alpha R}, \omega_j) = -c_j^{\alpha R} + c_0^{\alpha R}$ into the feasibility condition. □

Lemma 9. *There is a number* $\chi \in (1, 2]$ *such that, if* $\alpha_j < \chi$ *for all* $j \in J$*, the relaxed* *optimal tax problem has a unique solution with $\delta_j^{\alpha R} \in (\underline{\delta}, \bar{\delta})$ for all $j \in J$.*

Proof. As shown above, the FOC with respect to c_j is identical to

$$k_j(\delta_j^{\alpha R}, \alpha_j) = G_j(\delta_j^{\alpha R})(\alpha_j - 1) + g_j(\delta_j^{\alpha R})(\delta^*(\omega_j) - \delta_j^{\alpha R}) = 0 \,, \tag{31}$$

for every $j \in J$. Hence, the relaxed problem has a unique interior extremum for α if and only if $k_j(\delta, \alpha_j)$ has a unique root in δ with $\delta \in (\underline{\delta}, \bar{\delta})$. First, k_j is continuous in α_j and δ for all $\delta \in (\underline{\delta}, \bar{\delta})$. Second, $k_j(\underline{\delta}, \alpha_j) > 0$ for any $\alpha_j \geq 0$ because $\delta^*(\omega_j) \geq \delta^*(\omega_1) > \underline{\delta}$ for all $j \in J$ by (2) and the properties of h. Third, the derivative of k_j with respect to c_j (or its first argument) is given by

$$k_{j\delta}(\delta, \alpha_j) = g_j(\delta)\,(\alpha_j - 2) + g_j'(\delta)(\delta^*(\omega_j) - \delta) \,.$$

If k has a root at δ', this derivative has to be equal to

$$k_{j\delta}(\delta', \alpha_j) = G_j(\delta')\left[A_j(\delta')\,(\alpha_j - 2) - a_j(\delta')(\alpha_j - 1)\right] \,,$$

where $A_j(\delta) = \frac{g_j(\delta)}{G_j(\delta)}$ and $a_j(\delta) = \frac{g_j'(\delta)}{g_j(\delta)}$. Part (i) of Condition 1 ensures that $A_j(\delta) > a_j(\delta)$ for all $\delta \in \Delta$ and all $j \in J$. This implies that $k_{j\delta}(\alpha_j, \delta')$ is negative if

$$\alpha_j < \underline{\chi}_j(\delta') \coloneqq 1 + \frac{A_j(\delta')}{A_j(\delta') - a_j(\delta')} \,,$$

and that $\underline{\chi}_j(\delta') > 1$ for all $\delta' \in \Delta$ and all $j \in J$. Define χ_j as the minimum of $\underline{\chi}_j(\delta')$ over $\delta' \in (\underline{\delta}, \bar{\delta}\,]$. By assumption, $g_j(\delta)$ is larger than some number $\underline{g} > 0$ for all $\delta \in \Delta$ and $g_j'(\delta) \leq 0$ for some $\delta \in \Delta$. By the first property, χ_j is bounded away from 1 for all $j \in J$. By the second property, $\chi_j \leq 2$ for all $j \in J$. Consequently, $k_j(\delta, \alpha_j)$ has at most one root in δ if $\alpha_j < \chi_j$. If this root exists, it constitutes a maximum because $k_{j\delta}(\delta, \alpha_j) < 0$.

The existence of a root is ensured if, additionally, $k_j(\bar{\delta}, \alpha_j) < 0$, which is ensured for $\alpha_j < \chi_j' \coloneqq 1 + g_j(\bar{\delta})\left(\bar{\delta} - \delta^*(\omega_j)\right)$. Note that $\chi_j' > 1$ for all $j \in J$ by (2). Let χ be given by the minimum of χ_j and χ_j' over all $j \in J$. By the construction of χ, the relaxed problem has a unique interior extremum for α if $\alpha_j < \chi$ for all $j \in J$. □

Lemma 5 follows as a corollary from the Lemmas 8 and 9, using the definitions of labor supply distortions at both margins provided in Section 3.2. In particular, equation (15) implies that labor supply is undistorted at the intensive margin in all skill groups. Equation (16) implies that labor supply is downwards (upwards) distorted at the extensive margin in skill group $j \in J$ if and only if α_j is below (above) 1.

Proof of Lemma 6

Lemma 6 is proven through Lemmas 10 to 15.

Lemma 10. *For every $j \in J$ and every $\alpha \in \mathcal{A}^\chi$, $\delta_j^{\alpha R}$ is strictly increasing in α_j.*

Proof. Threshold $\delta_j^{\alpha R}$ is implicitly defined by condition (16) in Lemma 8. Using the implicit function theorem, its derivative with respect to α_j is given by

$$\frac{d\delta_j^{\alpha R}}{d\alpha_j} = \frac{A_j(\delta_j^{\alpha R})^{-1}}{1 + (1-\alpha_j)\left(1 - \frac{a_j(\delta_j^{\alpha R})}{A_j(\delta_j^{\alpha R})}\right)} = \frac{1}{(2-\alpha_j)A_j(\delta_j^{\alpha R}) - (1-\alpha_j)a_j(\delta_j^{\alpha R})}, \tag{32}$$

where the numerator equals $-k_\delta(\alpha_j, \delta_j^{\alpha R})/G_j(\hat{\delta}_j^{\alpha R}) > 0$ (see proof of Lemma 9). Hence, the derivative is strictly positive for all $\alpha_j < \chi$. □

Lemma 11. *For any skill group $j \in J_{-n}$, if $\alpha_j = \alpha_{j+1} = \alpha'$,*

(i) the difference $\delta_{j+1}^{\alpha R} - \delta_j^{\alpha R}$ is strictly increasing in α' for all $\alpha' \in (0, \chi)$ such that the downward IC constraint (29) *between skill groups j and $j+1$ is satisfied;*

(ii) $(c^{\alpha R}, y^{\alpha R})$ satisfies the downward IC constraint (29) *between skill groups j and $j+1$ for all $\alpha' \in [1, \chi)$;*

(iii) $(c^{\alpha R}, y^{\alpha R})$ satisfies the upward IC constraint between skill groups j and $j+1$,

$$\delta_{j+1} - \delta_j \leq h(y_{j+1}, \omega_j) - h(y_{j+1}, \omega_{j+1}) \ , \tag{33}$$

for all $\alpha' \in [0, 1]$.

Proof. (i) Using (32), the derivative of $\delta_{j+1}^{\alpha R} - \delta_j^{\alpha R}$ with respect to $\alpha_j = \alpha_{j+1} = \alpha'$ is strictly positive if

$$(2-\alpha')\left[A_j(\delta_j^{\alpha R}) - A_{j+1}(\delta_{j+1}^{\alpha R})\right] > (1-\alpha')\left[a_j(\delta_j^{\alpha R}) - a_{j+1}(\delta_{j+1}^{\alpha R})\right] \ . \tag{34}$$

In all allocations that satisfy the downward IC constraint, $\delta_{j+1}^{\alpha R} > \delta_j^{\alpha R}$. By Conditions 1 and 2, we hence have $A_j(\delta_j^{\alpha R}) - A_{j+1}(\delta_{j+1}^{\alpha R}) > 0$ and $a_j(\delta_j^{\alpha R}) - a_{j+1}(\delta_{j+1}^{\alpha R}) \geq 0$. Because $\chi \in (1, 2]$, this directly implies that inequality (34) is satisfied for any $\alpha' \in [1, \chi)$. Moreover, Condition 2 ensures that $a_j(\delta_j^{\alpha R}) - a_{j+1}(\delta_{j+1}^{\alpha R}) \leq 2\left[A_j(\delta_j^{\alpha R}) - A_{j+1}(\delta_{j+1}^{\alpha R})\right]$. Hence, inequality (34) is also satisfied for the alternative case $\alpha' \in (0, 1)$, where $(2-\alpha')/(1-\alpha') > 2$.

(ii) For $\alpha_j = 1$, $\delta_k^{\alpha R} = \delta^*(\omega_k)$ for $k \in \{j, j+1\}$. Inserting these into the downward IC constrait (29) and rearranging terms gives

$$\delta^*(\omega_{j+1}) = y_{j+1}^{\alpha R} - h\left(y_{j+1}^{\alpha R}, \omega_{j+1}\right) \geq y_j^{\alpha R} - h\left(y_j^{\alpha R}, \omega_{j+1}\right) \ ,$$

which holds with strict inequality by the single-crossing property. The left-hand side of (29) is given by $\delta_{j+1}^{\alpha R} - \delta_j^{\alpha R}$, which is strictly increasing in α' by part (i), while the right-hand side is constant. Hence, the downward IC constraint is satisfied for all $\alpha' \geq 1$.

(iii) For $\alpha_j = 1$, inserting $\delta_k^{\alpha R} = \delta^*(\omega_k)$ for $k \in \{j, j+1\}$ into the upward IC constraint (33) gives $y_{j+1}^{\alpha R} - h\left(y_{j+1}^{\alpha R}, \omega_j\right) \leq \delta^*(\omega_j) = \max_{y>0}\{y - h(y, \omega_j)\}$, which is again satisfied with strict inequality. As the left-hand side of (33) is strictly increasing in α', the upward IC is satisfied for all $\alpha' < 1$ such that the downward IC is satisfied. As the downward and upward IC constraints cannot be violated at the same time, the upward IC constrained is also satisfied for all other $\alpha' < 1$.

□

Lemma 12. *Consider the downward IC constraint* (29) *between the workers with skill types ω_j and $\omega_{j+1} = a_j\omega_j$ for some $j \in J_{-n}$. Define a_j^0 as the supremum of the set of real numbers $a' > 1$ such that, given $\alpha_j = \alpha_{j+1} = 0$, $(c^{\alpha R}, y^{\alpha R})$ violates the downward IC constraint for all $a_j \in (1, a')$.*

(i) If Condition 1 is satisfied, $a_j^0 > 1$ exists.

(ii) Let $\alpha_j = \alpha_{j+1} = \alpha'$. For each $a_j \in \left(1, a_j^0\right)$, there exists a unique number $\underline{\beta}_j \in (0,1)$ such that (c_α, y_α) violates the downward IC constraint (20) *if and only if $\alpha' < \underline{\beta}_j$.*

Proof. (i) In the limit case $a_j = 1$, the downward IC constraint (29) simplifies to $\delta_{j+1}^{\alpha R} - \delta_j^{\alpha R} \geq 0$. By (16)$\delta_k^{\alpha R}$ is implicitly defined by $\frac{1}{A_k(\delta_k^{\alpha R})} + \delta_k^{\alpha R} = \delta^*(\omega_k)$ for $\alpha' = 0$ and any $k \in J$. For $a_j = 1$, we have $\delta^*(\omega_{j+1}) = \delta^*(\omega_j)$. By Condition 1 (ii), $A_j(\delta) \geq A_{j+1}(\delta)$. Hence, there are two possible cases. First, if $A_j(\delta_j^{\alpha R}) > A_{j+1}(\delta_j^{\alpha R})$, we have $\delta_{j+1}^{\alpha R} < \delta_j^{\alpha R}$. Hence, the downward IC constraint is violated for $\alpha' = 0$ and $a = 1$. Second, if $A_j(\delta_j^{\alpha R}) = A_{j+1}(\delta_j^{\alpha R})$, we have $\delta_{j+1}^{\alpha R} = \delta_j^{\alpha R}$. Hence, the downward IC constraint is satisfied with equality for $\alpha' = 0$ and $a = 1$.

To consider the general case $a_j > 1$, I compare the derivatives of both sides of (29) with respect to a_j at $a_j = 1$. The derivative of the left-hand side is given by

$$\frac{\frac{d\delta^*(\omega_{j+1})}{da_j}}{2 - \frac{a_{j+1}(\delta_{j+1}^{\alpha R})}{A_{j+1}(\delta_{j+1}^{\alpha R})}} = \frac{\frac{d\left[y_{j+1}^{\alpha R} - h(y_{j+1}^{\alpha R}, \omega_j a_j)\right]}{da_j}}{2 - \frac{a_{j+1}(\delta_{j+1}^{\alpha R})}{A_{j+1}(\delta_{j+1}^{\alpha R})}} = -\frac{\omega_j h_\omega\left(y_{j+1}^{\alpha R}, \omega_j a\right)}{2 - \frac{a_{j+1}(\delta_{j+1}^{\alpha R})}{A_{j+1}(\delta_{j+1}^{\alpha R})}} > 0\ ,$$

where the term in the denominator is strictly larger than 1 because $a_{j+1}(\delta) < A_{j+1}(\delta)$ for all $\delta \in \Delta$ by Condition 1 (i). The derivative of the right-hand side is given by $-\omega_j h_\omega\left(y_j^{\alpha R}, \omega_j a\right) > 0$, which is strictly larger than the derivative of the left-hand side. Hence, the downward IC constraint is unambiguously violated for $\alpha_j = \alpha_{j+1} = 0$ and all a_j between 1 and some number $a' > 1$. The supremum a_j^0 is hence well-defined, and may either be given by a finite number or by ∞.

(ii) For any $a_j \in \left(1, a_j^0\right)$, the right-hand side of (29) is independent of α'. The left-hand side $\delta_{j+1}^{\alpha R} - \delta_j^{\alpha R}$ is small enough to violate the downward IC constraint for $\alpha' = 0$ by part (i), and large enough to satisfy this constraint with strict inequality for $\alpha' = 1$ by Lemma 11 (ii). By Lemma 11 (i), $\delta_{j+1}^{\alpha R} - \delta_j^{\alpha R}$ is strictly increasing in α' for all levels of α' such that (29) is satisfied. Consequently, there exists a unique threshold $\underline{\beta}_j \in (0,1)$ such that the downward IC constraint is violated for all $\alpha' \in \left[0, \underline{\beta}_j\right)$ and satisfied for all $\alpha' \in \left[\underline{\beta}_j, \chi\right)$.

□

Lemma 13. *Consider the upward IC constraint* (18) *between the workers with skill types ω_j and $\omega_{j+1} = a_j\omega_j$ for some $j \in J_{-n}$. Define $a_j^u(\gamma)$ as the supremum of the set of real numbers $a' > 1$ such that, given $\alpha_j = \alpha_{j+1} = \gamma > 1$, $(c^{\alpha R}, y^{\alpha R})$ violates the upward IC constraint for all $a_j \in (1, a')$.*

(i) If Condition 1 is satisfied, $a_j^u(\gamma)$ exists for all $\gamma \in (1, \chi)$.

(ii) Let $\alpha_j = \alpha_{j+1} = \alpha'$. For each $\gamma \in (1,\chi)$ and each $a \in \left(1, a_j^u(\gamma)\right)$, there exists a unique number $\bar{\beta}_j \in (1, \gamma)$ such that (c_α, y_α) violates the upward IC constraint if and only if $\alpha' \in \left(\bar{\beta}_j, \chi\right)$.

Proof. (i) Fix a number $\gamma \in (1,\chi)$. In the limit case $a_j = 1$, the upward IC constraint (33) simplifies to $\delta_{j+1}^{\alpha R} - \delta_j^{\alpha R} \leq 0$. For $\alpha_j = \alpha_{j+1} = \gamma > 1$, $\delta_k^{\alpha R}$ is implicitly defined by $\frac{1-\gamma}{A_k(\delta_k^{\alpha R})} + \delta_k^{\alpha R} = \delta^*(\omega_k)$ for $k \in \{j, j+1\}$. For $a = 1$, $\delta^*(\omega_j) = \delta^*(\omega_{j+1})$. By Condition 1 (ii), $A_j(\delta) \geq A_{j+1}(\delta)$ for all $\delta \in \Delta$. Again, there are two possible cases. First, if $A_j(\delta_j^{\alpha R}) > A_{j+1}(\delta_j^{\alpha R})$, we have $\delta_{j+1}^{\alpha R} > \delta_j^{\alpha R}$ so that (33) is violated. Second, if $A_j(\delta_j^{\alpha R}) = A_{j+1}(\delta_j^{\alpha R})$, we have $\delta_{j+1}^{\alpha R} = \delta_j^{\alpha R}$ so that (33) is satisfied with strict equality.

To consider the general case $a_j > 1$, I again compare the derivatives of both sides of (33) in a_j, given a fixed γ. The derivative of the left-hand side is given by

$$\frac{-\omega_j h_\omega\left(y_{j+1}^{\alpha R}, a_j\omega_j\right)}{1-(\gamma-1)\left(1-\frac{a_{j+1}(\delta_{j+1}^{\alpha R})}{A_{j+1}(\delta_{j+1}^{\alpha R})}\right)} > 0\,,$$

where the term in the denominator is strictly positive for any $\gamma < \chi$ and strictly smaller than 1 by Condition 1 (i) for all $\gamma > 1$. The derivative of the right-hand side is given by

$$-\omega_j h_\omega\left(y_{j+1}^{\alpha R}, a_j\omega_j\right) + \left[h(y_{j+1}^{\alpha R}, \omega_j) - h(y_{j+1}^{\alpha R}, a_j\omega_j)\right]\frac{dy_{j+1}^{\alpha R}}{da_j} > 0\,.$$

At $a_j = 1$, the term in squared brackets equals zero. Hence, the derivative of the left-hand side is strictly larger than the derivative of the right-hand side of (33). Thus, the upward CI constraint is violated for $\alpha_j = \alpha_{j+1} = \gamma > 1$ and all a_j between 1 and some number $a' > 1$. The supremum $a_j^u(\gamma)$ is hence well-defined, and may either be given by some finite number above 1 or by ∞.

(ii) Fix any $\gamma \in (1, \chi)$. If $a_j \in \left(1, a_j^u(\gamma)\right)$, the difference $\delta_{j+1}^{\alpha R} - \delta_j^{\alpha R}$ is large enough to violate (33) for $\alpha' = \gamma$ and small enough to satisfy (33) for $\alpha' = 1$. Moreover, $\delta_{j+1}^{\alpha R} - \delta_j^{\alpha R}$ is strictly increasing in α' for all $\alpha' > 1$ by Lemma 11 (iii) and (i). Consequently, there exists a unique threshold $\bar{\beta}_j \in (1, \gamma)$ such that the downward IC constraint is violated for all $\alpha' \in \left(\bar{\beta}_j, \chi\right)$, and satisfied for all $\alpha' \in \left[0, \bar{\beta}_j\right]$.

□

Lemma 14. *Consider any $j \in J_{-n}$. If $a_j < a_j^0$, there is a continuously differentiable and strictly increasing function $\beta_j^D : [0, \chi] \to (0, \chi)$ such that $(c^{\alpha R}, y^{\alpha R})$ satisfies the downward IC constraint if and only if $\alpha_{j+1} \geq \beta_j^D(\alpha_j)$. Function β_j^D has a unique fixed point at $\underline{\beta}_j \in (0, 1)$, where $\frac{d\beta_j^D(\underline{\beta}_j)}{d\alpha_j} < 1$.*

Proof. Consider some $j \in J_{-n}$ and some $a_j \in \left(1, a_j^0\right)$, where a_j^0 is defined in Lemma 12 (i). Hence, there is a number $\underline{\beta}_j \in (0, 1)$ such that $(c^{\alpha R}, y^{\alpha R})$ satisfies the downward IC constraint (29) if $\alpha_j = \alpha_{j+1} \geq \underline{\beta}_j$, and violates (29) if $\alpha_j = \alpha_{j+1} < \underline{\beta}_j$. Recall that $\delta_k^{\alpha R}$ is strictly increasing in α_k for each $k \in J$ and each $\alpha_k \in [0, \chi)$ by Lemma 10 and that changes in α_j and α_{j+1} only affect the left-hand side of the downward IC (29).

First, fix some $\alpha_j = x \in \left(\underline{\beta}_j, \chi\right)$. For $\alpha_{j+1} = \underline{\beta}_j$, the difference $\delta_{j+1}^{\alpha R} - \delta_j^{\alpha R}$ is smaller than for $\alpha_j = \alpha_{j+1} = \underline{\beta}_j$, and (29) is violated. For $\alpha_{j+1} = \alpha_j = x$, in contrast, (29) is satisfied by Lemma 12. As $\delta_{j+1}^{\alpha R}$ is increasing in α_{j+1}, there is a unique number $\beta_1^D(x) \in \left(\underline{\beta}, x\right)$ such that (29) is satisfied for all $\alpha_{j+1} \geq \beta_j^D(x)$, and violated for all $\alpha_{j+1} < \beta_j^D(x)$.

Second, fix some $\alpha_j = x \in \left[0, , \underline{\beta}_j\right)$. For $\alpha_{j+1} = \underline{\beta}$, the difference $\delta_{j+1}^{\alpha R} - \delta_j^{\alpha R}$ is larger than for $\alpha_j = \alpha_{j+1} = \underline{\beta}_j$ so that (29) is satisfied. For $\alpha_{j+1} = x$, in contrast, (29) is violated by Lemma 12. By the monotonicity of $\beta_{j+1}^{\alpha R}$ in α_{j+1}, there is a unique number $\beta_j^D(x) \in \left(x, \underline{\beta}_j\right)$ such that (29) is satisfied if $\alpha_{j+1} \geq \beta_j^D(x)$, and violated if $\alpha_{j+1} < \beta_j^D(x)$. This also implies that Hence, $\beta_1^D(x) \in (0, \chi)$ for all $x \in [0, \chi)$.

To prove that β_j^D is continuously differentiable and strictly increasing, note that $\beta_j^D(\alpha_j)$ is implicitly defined by

$$\delta_{j+1}^{\alpha R}\left(\beta_j^D(\alpha_j)\right) - \delta_j^{\alpha R}(\alpha_j) = h\left(y_j^{\alpha R}, \omega_j\right) - h\left(y_j^{\alpha R}, \omega_{j+1}\right) . \tag{35}$$

The right-hand side of this equation is constant in α_j and α_{j+1}. The left-hand side is continuously differentiable and monotonic in both weights as long as these are below χ. Hence, β_j^D is continuously differentiable in α_j. Using the implicit function theorem, the derivative is given by

$$\frac{d\beta_j^D(\alpha_j)}{d\alpha_j} = \frac{\frac{d\delta_j^{\alpha R}(\alpha_j)}{d\alpha_j}}{\frac{d\delta_{j+1}^{\alpha R}(\beta_j^D(\alpha_j))}{d\alpha_{j+1}}} > 0 , \tag{36}$$

where the numerator and the denominator are strictly positive by Lemma 10.

Finally, we know from Lemma 12 that β_j^D has a unique fixed point at $\underline{\beta}_j < 1$ whenever $a_j \in \left(1, a_j^0\right)$. As the downward IC constraint is satisfied at this fixed point, Lemma 11 (i) ensures that $\frac{d\delta_j^{\alpha R}(\underline{\beta}_j)}{d\alpha_j} < \frac{d\delta_{j+1}^{\alpha R}(\underline{\beta})}{d\alpha_{j+1}}$. Hence, $\frac{d\beta_j^D(\underline{\beta}_j)}{d\alpha_j} < 1$. □

Lemma 15. *Consider any $j \in J_{-n}$. If $a_j < a_j^u(\gamma)$ for some $\gamma \in (1, \chi)$, there is a continuously differentiable and strictly increasing function $\beta_j^U : [0, \chi) \to (0, \chi_n)$ such that $(c^{\alpha R}, y^{\alpha R})$ violates the upward IC constraint if and only if $\alpha_{j+1} > \beta_j^U(\alpha_j)$. Function β_j^U has a unique fixed point at $\bar{\beta}_j \in (1, \chi)$, where $\frac{d\beta_j^U(\underline{\beta}_j)}{d\alpha_j} < 1$.*

Proof. The proof of Lemma 15 follows the same steps as the proof of Lemma 14. Consider some $j \in J_{-n}$ and some a_j between 1 and the treshold $a_j^u(\gamma)$ for some $\gamma \in (1, \chi)$, as defined in Lemma 13 (i). Hence, there is a number $\bar{\beta}_j \in (1, \gamma)$ such that $(c^{\alpha R}, y^{\alpha R})$ satisfies the upward IC constraint (33) if $\alpha_j = \alpha_{j+1} \geq \bar{\beta}_j$, and violates (33) if $\alpha_j = \alpha_{j+1} < \bar{\beta}_j$.

First, fix some $\alpha_j = x \in (\bar{\beta}_j, \chi)$. The upward IC constraint is satisfied for $\alpha_{j+1} = \bar{\beta}_j$, and violated $\alpha_{j+1} = x$ by Lemma 13. By the monotonicity of $\delta_{j+1}^{\alpha R}$ in α_{j+1}, there is a unique number $\beta_1^U(x) \in (\bar{\beta}_j, x)$ such that (33) is satisfied if $\alpha_{j+1} \leq \beta_j^U(x)$, and violated if $\alpha_{j+1} > \beta_j^U(x)$.

Second, fix some $\alpha_j = x \in [0, \bar{\beta}_j)$. The upward IC constraint is violated for $\alpha_{j+1} = \bar{\beta}_j$, and satisfied for $\alpha_{j+1} = x$ by Lemma 13. Hence, there is a unique number $\beta_j^U(x) \in (x, \bar{\beta}_j)$ such that (33) is satisfied if $\alpha_{j+1} \leq \beta_j^U(x)$ and violated if $\alpha_{j+1} > \beta_j^U(x)$. Hence, we also have $\beta_j^U(x) \in (0, \chi)$ for all $x \in [0, \chi)$.

For each $\alpha_j \in [0, \chi)$, $\beta_j^U(\alpha_j)$ is implicitly defined by

$$\delta_{j+1}^{\alpha R}\left(\beta_j^U(\alpha_j)\right) - \delta_j^{\alpha R}(\alpha_j) = h\left(y_{j+1}^{\alpha R}, \omega_j\right) - h\left(y_{j+1}^{\alpha R}, \omega_{j+1}\right) , \tag{37}$$

where the left-hand side is continuously differentiable and monotonic in α_j and α_{j+1}, and the right-hand side is constant in both social weights. Using the implicit function theorem, the derivative of β_j^U with respect to α_j is given by

$$\frac{d\beta_j^U(\alpha_j)}{d\alpha_j} = \frac{\frac{d\delta_j^{\alpha R}(\alpha_j)}{d\alpha_j}}{\frac{d\delta_{j+1}^{\alpha R}(\beta_j^U(\alpha_j))}{d\alpha_{j+1}}} > 0 , \tag{38}$$

where the numerator and the denominator are strictly positive by Lemma 10. This derivative is continuous and strictly positive for all $\alpha_1 \in [0, \chi)$.

Finally, Lemma 13 implies that β_j^U has a unique fixed point at $\bar{\beta}_j \in (1, \chi)$ whenever $a_j \in \left(1, a_j^U(\gamma)\right)$ for some $\gamma \in (1, \chi)$. At this fixed point, the downward IC constraint is satisfied and $\frac{d\delta_j^{\alpha R}(\bar{\beta}_j)}{d\alpha_j} < \frac{d\delta_{j+1}^{\alpha R}(\bar{\beta}_j)}{d\alpha_{j+1}}$ by Lemma 11 (i). Consequently, $\frac{d\beta_j^U(\bar{\beta}_j)}{d\alpha_j} < 1$. □

Finally, this allows us to prove Lemma 6.

Proof. Define $a_j^{Us} := \sup\left\{a_j^U(\gamma) \,|\, \gamma \in (1, \chi)\right\}$. For all $a_j < \min\left\{a_j^0, a_j^{Us}\right\}$, Lemmas 14 and 15 directly ensure the existence of two functions β_j^D and β_j^U with the properties stated in Lemma 6.

It only remains to prove that $\beta_j^U(\alpha_j) > \beta_j^D(\alpha_j)$ for all $\alpha_j \in [0,\chi)$. Note that $h\left(y_{j+1}^{\alpha R},\omega_j\right) - h\left(y_{j+1}^{\alpha R},\omega_{j+1}\right) > h\left(y_j^{\alpha R},\omega_j\right) - h\left(y_j^{\alpha R},\omega_{j+1}\right)$ by the properties of the effort cost function h. Hence, equations (35) and (37) imply that $\delta_{j+1}^{\alpha R}\left(\beta_j^U(x)\right) > \delta_{j+1}^{\alpha R}\left(\beta_j^D(x)\right)$. Because $\delta_{j+1}^{\alpha R}$ is strictly increasing in α_{j+1} by Lemma 10, we have $\delta_j^U(x) > \delta_j^D(x)$ for all $x \in [0,\chi)$. □

Proof of Proposition 1

I prove Proposition 1 through Lemmas 16 to 21, which identify properties of the optimal allocation (c^α, y^α) for different sets of binding IC constraints along the skill dimension. To refer to these different constellations, I will henceforth say that two skill groups j and k are downwards-linked (upwards-linked) if all downward (upward) IC constraints between the pairs $(j,j+1)$, ..., $(k-1,k)$ are binding.

Lemma 16. *For each $j \in J$, output y_j^α and participation threshold δ_j^α satisfy the conditions*

$$\delta_j^\alpha = y_j^\alpha - h(y_j^\alpha,\omega_j) + \frac{\alpha_j - 1}{A_j(\delta_j^\alpha)} + \frac{\nu_{j-1}^D - \nu_j^D - \nu_{j-1}^U + \nu_j^U}{f_j g_j(\delta_j^\alpha)}\,, \tag{39}$$

$$1 - h_y(y_j,\omega_j) = \frac{h_y(y_j,\omega_j) - h_y(y_j,\omega_{j+1})}{f_j G_j(\delta_j^\alpha)}\nu_j^D - \frac{h_y(y_j,\omega_{j-1}) - h_y(y_j,\omega_j)}{f_j G_j(\delta_j^\alpha)}\nu_{j-1}^U\,, \tag{40}$$

where ν_k^D and ν_k^U denote the Lagrange multipliers associated with the downward IC (20) *and the upward IC* (18)*, respectively, between the workers with skill types ω_k and ω_{k+1}.*

Proof. The Lagrangian of the optimal tax problem is given by

$$\begin{aligned}
\mathcal{L} = & \sum_{j=1}^n f_j\Big(\int_{\underline{\delta}}^{\delta_j} g_j(\delta)\gamma(\omega,\delta)\Psi\left[c_j - h(y_j,\omega_j) - \delta\right] d\delta + \int_{\delta_j}^{\bar{\delta}} g_j(\delta)\gamma(\omega,\delta)\Psi(c_0)\, d\delta\Big) \\
& + \lambda\left[\sum_{j=1}^n f_j G_j(\delta_j)(y_j - c_j + c_0) - c_0\right] \\
& + \sum_{j=1}^{n-1}\nu_j^D\left[c_{j+1} - h(y_{j+1},\omega_{j+1}) - c_j + h(y_j,\omega_{j+1})\right] \\
& + \sum_{j=1}^{n-1}\nu_j^U\left[c_j - h(y_j,\omega_j) - c_{j+1} + h(y_{j+1},\omega_j)\right]\,.
\end{aligned}$$

For any $j \in \{2,\dots,n-1\}$, the FOCs with respect to c_j and y_j are given by:

$$\begin{aligned}
\mathcal{L}_{c_j} = & f_j\left[G_j(\delta_j^\alpha)\left(\alpha_j - \lambda\right) + \lambda g_j(\delta_j^\alpha)(y_j^\alpha - c_j^\alpha + c_0^\alpha)\right] + \nu_{j-1}^D - \nu_j^D - \nu_{j-1}^U + \nu_j^U \quad \stackrel{!}{=} 0 \\
\mathcal{L}_{y_j} = & f_j\left[-h_y(y_j^\alpha,\omega_j)\Big(G_j(\delta_j^\alpha)\alpha_j + \lambda g_j(\delta_j^\alpha)(y_j^\alpha - c_j^\alpha + c_0^\alpha)\Big) + \lambda G_j(\delta_j^\alpha)\right] \\
& - \left(\nu_{j-1}^D + \nu_j^U\right)h_y(y_j^\alpha,\omega_j) + \nu_j^D h_y(y_j^\alpha,\omega_{j+1}) + \nu_{j-1}^U h_y(y_j^\alpha,\omega_{j-1}) \quad \stackrel{!}{=} 0
\end{aligned}$$

The FOC with respect to c_0 is identical to the one for the relaxed problem (see Lemma 8). Combining the FOCs with respect to c_j for all $j \in \{0, 1, \ldots, n\}$ gives $\lambda = 1$. Equation (39) then follows from rearranging the FOC with respect to c_j. Combining the FOCs with respect c_j and y_j gives equation (40).

Below, I will exploit that, setting $\nu_j^U = \nu_j^D = 0$, Lemma 16 also provides the conditions for the solution to a relaxed problem that ignores the local IC constraints between the workers with skill types j and $j+1$. $\square$

Lemma 17. *For any $\alpha \in \mathcal{A}^{\chi}$, if k is the lowest skill group that is upwards-linked with $l > k$, then*

(i) $\alpha_k > 1$,

(ii) $\delta_j^{\alpha} > y_j^{\alpha} - h(y_j^{\alpha}, \omega_j)$ for all $j \in \{k, \ldots, l\}$, and

(iii) $\alpha_j > 1$ for all $j \in \{1, \ldots, l\}$.

Proof. For part i, note first that k and $l > k$ cannot be upwards-linked unless either $\alpha_{j+1} > \alpha_j(\beta_j^U)$ for at least one $j \in \{k, \ldots, l-1\}$ or the downward IC between l and $l+1$ is binding, $\nu_l^D > 0$. Assume both conditions would be violated, and consider a relaxed problem where all local ICs between k and l are ignored. In the solution to this problem, $\delta_k \geq \delta_k^{\alpha R}$, $\delta_j = \delta_j^{\alpha R}$ for all $j \in \{k+1, \ldots, l\}$ and $y_j = y_j^{\alpha R}$ for all $j \in \{k, \ldots, l\}$. Hence, the solution to this relaxed problem satisfies all upward IC constraints between k and l. Consequently, these upward IC constraints cannot be binding in (c^{α}, y^{α}).

Second, assume that there is a skill group $j \in \{k, \ldots, l-1\}$ such that $\alpha_{j+1} > \alpha_j(\beta_j^U)$. By Lemma 6, $\alpha_j > \bar{\beta}_j > 1$. For the monotonicity of $\alpha \in \mathcal{A}$, this ensures that $\alpha_k > 1$.

Third, assume that the upward IC between $l-1$ and l as well as the downward IC between l and $l+1$ are binding and that $\alpha_{j+1} \leq \alpha_j(\beta_j^U)$ for all $j \in \{k, \ldots, l-1\}$. Consider a relaxed problem where only the local ICs between skill groups $l-1$ and l are ignored. In the solution to this relaxed problem, the downward IC between l and $l+1$ has to be binding as well. From Lemma 16, the solution of this problem satisfies $y_{l-1} = y_{l-1}^{\alpha R}$, $\delta_{l-1} \geq \delta^*(\omega_{l-1}) + (\alpha_{l-1} - 1)/A_{l-1}(\delta_{l-1})$ and $\delta_l < y_l - h(y_l, \omega_l) + (\alpha_l - 1)/A_l(\delta_l)$. By $\alpha_l < \alpha_{l-1}$, this allocation has to satisfy

$$\delta_l - \delta_{l-1} < \left(\frac{1}{A_l(\delta_l)} - \frac{1}{A_{l-1}(\delta_{l-1})} \right) (\alpha_{l-1} - 1) + y_l - h(y_l, \omega_l) - \delta^*(\omega_{l-1}) \, .$$

The allocation only violates the upward IC constraint (33) if $\delta_l - \delta_{l-1} > h(y_l, \omega_{l-1}) - h(y_l, \omega_l)$. Hence, this can only result if

$$\left(\frac{1}{A_l(\delta_l^{\alpha P})} - \frac{1}{A_{l-1}(\delta_{l-1}^{\alpha P})} \right) (\alpha_{l-1} - 1) > \delta^*(\omega_{l-1}) - \left[y_l^{\alpha P} - h(y_l^{\alpha P}, \omega_{l-1}) \right] \geq 0 \, ,$$

where the last inequality follows by the definition of $\delta^*(\omega_{l-1})$. Recall that $A_l(\delta_l^{\alpha P}) <$

$A_{l-1}(\delta_{l-1}^{\alpha P})$ by Lemma 2. Hence, the condition above can only be satisfied if α_{l-1} is strictly larger than 1. But this implies that $\alpha_k > 1$ for all $\alpha \in \mathcal{A}^X$.

For part ii, note that k is by construction the lowest skill group that is upwards-linked to l, i.e., the upward IC between $k-1$ and k is not binding. Hence, $\delta_k^\alpha > y_k^\alpha - h(y_k^\alpha, \omega_k) + \frac{\alpha_k - 1}{A_k(\delta_k^\alpha)} > y_k^\alpha - h(y_k^\alpha, \omega_k)$ because $\alpha_k > 1$ as shown in part (i). By Lemma 16, we have $y_j^\alpha > y_j^{\alpha R}$ for all $j \in \{k, \ldots, l-1\}$ because the corresponding upward IC constraints are binding, i.e., $\nu_j^U > 0$. As usual, incentive compatibility also requires $y_{j+1}^\alpha > y_j^\alpha$. The single-crossing condition hence ensures that, for all $j \in \{k, \ldots, l-1\}$, $y_j^\alpha - h(y_j^\alpha, \omega_j) > y_{j+1}^\alpha - h(y_{j+1}^\alpha, \omega_j)$ and

$$\begin{aligned}\delta_{j+1}^\alpha &= \delta_j^\alpha + h(y_{j+1}^\alpha, \omega_j) - h(y_{j+1}^\alpha, \omega_{j+1}) \\ &> y_j^\alpha - h(y_j^\alpha, \omega_j) + h(y_{j+1}^\alpha, \omega_j) - h(y_{j+1}^\alpha, \omega_{j+1}) \geq y_{j+1}^\alpha - h(y_{j+1}^\alpha, \omega_{j+1}) \;.\end{aligned}$$

For part iii, finally, assume that m is the highest skill group that is upwards-linked with k. By the previous arguments, $\delta_m^\alpha > y_m^\alpha - h(y_m^\alpha, \omega_m)$. At the same time, $\delta_m^\alpha < y_m^\alpha - h(y_m^\alpha, \omega_m) + (\alpha_m - 1)/A_m(\delta_m^\alpha)$ by equation (39). Both statements can only be consistent if $\alpha_m > 1$. For all $\alpha \in \mathcal{A}^X$, we hence have $\alpha_j > 1$ for all $j \in \{1, \ldots, m\}$. □

Lemma 18. *For any $\alpha \in \mathcal{A}^X$, if the skill groups j and $j+1$ are downwards-linked and $\delta_j^\alpha \leq y_j^\alpha - h(y_j^\alpha, \omega_j)$, then (c^α, y^α) also involves $\delta_{j+1}^\alpha \leq y_{j+1}^\alpha - h(y_{j+1}^\alpha, \omega_j)$.*

Proof. If the downward IC constraint (29) is binding, $\delta_{j+1} = \delta_j + h(y_j, \omega_j) - h(y_j, \omega_{j+1})$. Hence, we have $\delta_{j+1} \leq y_j^\alpha - h(y_j^\alpha, \omega_{j+1}) \leq y_{j+1}^\alpha - h(y_{j+1}^\alpha, \omega_{j+1})$. The second inequality results because $y_j \leq y_{j+1}$ in every incentive-compatible allocation and $y_{j+1}^\alpha \leq \arg\max_{y>0} y - h(y, \omega_{j+1})$ if the skill groups j and $j+1$ are downwards-linked. □

Lemma 19. *For any $\alpha \in \mathcal{A}^X$, there exists a number $k^\alpha \in (0, n]$ such that labor supply in skill group j is upwards distorted at the extensive margin if and only if $j \in \{l \in J : l \leq k^\alpha\}$.*

Proof. Fix some $\alpha \in \mathcal{A}^X$. Consider a skill group j for which labor supply in (c^α, y^α) is not upwards distorted at the extensive margin, $\delta_j^\alpha \leq y_j^\alpha - h(y_j^\alpha, \omega_j)$. As I will show, this ensures that optimal labor supply is not upwards distorted at the extensive margin in any skill group $h > j$ as well.

By Lemma 17, j cannot be upwards-linked to other skill groups. Let l be the highest skill group to which j is downwards-linked. (Note that l equals j if the downward IC between skill groups j and $j+1$ is slack.) By Lemma 18, $\delta_k^\alpha \leq y_k^\alpha - h(y_k^\alpha, \omega_k)$ for all $k \in \{j, \ldots, l\}$. By Lemma 16, we must at the same time have $\delta_l^\alpha \geq y_l^\alpha - h(y_l^\alpha, \omega_l) + (\alpha_l - 1)/A_l(\delta_l^\alpha)$. Hence, we must have $\alpha_l \leq 1$.

Consequently, there cannot be upwards-linked skill groups above l by Lemma 16. For any unlinked skill group $k > l$, $\alpha_k < \alpha_l \leq 1$ ensures that labor supply is not upwards distorted at the extensive margin. For any downwards-linked skill groups $k > l$ and m, $\alpha_k < 1$ and $\nu_k^D > 0$ jointly ensure that $\delta_k < y_k^\alpha - h(y_k^\alpha, \omega_k)$. By Lemma 18, the same conditions holds for the skill groups $k + 1$ to m. Hence, labor supply is not upwards distorted at the extensive margin in any skill group $h > j$. □

Lemma 20. *Consider some weight $\alpha \in \mathcal{A}^{\chi}$ such that the optimal tax problem has an interior solution (c^{α}, y^{α}). Then,*

(i) the consumption level c_0^{α} of the unemployed is strictly positive;

(ii) there is a number $k^{\alpha} \in (0, n)$ such that optimal output is

(a) upwards distorted at the extensive margin in skill group j if and only if $j \leq k^{\alpha}$, and

(b) downwards distorted or undistorted at the intensive margin in skill group j for all $j \geq k^{\alpha}$;

(iii) optimal output in the highest skill group n is undistorted at the intensive margin and downwards distorted at the extensive margin.

Proof. (i) Consider some implementable and Pareto-efficient allocation (c, y) with $c_0 \leq 0$. In this allocation, the feasibility condition must hold with equality, so that $c_0 = \sum_{j=1}^{n} f_j G_j(\delta_j)(y_j - c_j + c_0) \leq 0$. Moreover, an increase in c_0 must not self-financing. I will show that there exists a marginal variation that is feasible, incentive-compatible and welfare-increasing given any weight sequence $\alpha \in \mathcal{A}^{\chi}$.

Consider an allocation $(\tilde{c}, y)$ with $\tilde{c}_j = c_j - \varepsilon$ for all $j \in J$ and $\tilde{c}_0$ chosen to balance the feasibility condition. In particular, consider a marginal increase in ε from $\varepsilon = 0$. This leaves the IC constraints between all workers satisfied, but induces some workers in all skill groups to leave the labor force. Taking these responses into account, we have

$$\left.\frac{d\tilde{c}_0}{d\varepsilon}\right|_{\varepsilon=0} = \frac{\sum_{j=1}^{n} f_j G_j(\delta_j) - \sum_{j=1}^{n} f_j G_j(\delta_j) Z_j}{1 - \sum_{j=1}^{n} f_j G_j(\delta_j) + \sum_{j=1}^{n} f_j G_j(\delta_j) Z_j} ,$$

where the denominator is strictly positive (otherwise, an increase in c_0 would be self-financing) and $Z_j = \frac{g_j(\delta_j)}{G_j(\delta_j)} (y_j - c_j + c_0) = \frac{g_j(\delta_j)}{G_j(\delta_j)} (y_j - h(y_j, \omega_j) - \delta_j)$. If allocation (c, y) is welfare-optimal, there is a unique number $k \in (0, n]$ such that $Z_j < 0$ for all $j < k$ and $Z_j \geq 0$ for all $j \geq k$ by Lemma 18. Moreover, $\frac{g_j(\delta_j)}{G_j(\delta_j)} = A_j(\delta_j) > A_k(\delta_k)$ for all $j < k$ and $\frac{g_j(\delta_j)}{G_j(\delta_j)} < A_k(\delta_k)$ for all $j > k$ by Lemma 2. Hence, we have

$$\sum_{j=1}^{n} f_j G_j(\delta_j) Z_j < A_k(\delta_k) \sum_{j=1}^{n} f_j G_j(\delta_j) (y_j - c_j + c_0) = A_k(\delta_k) c_0 \leq 0 ,$$

which implies that $\left.\frac{d\tilde{c}_0}{d\varepsilon}\right|_{\varepsilon=0} > \frac{\sum_{j=1}^{n} f_j G_j(\delta_j)}{1 - \sum_{j=1}^{n} f_j G_j(\delta_j)}$. The marginal welfare effect of increasing ε follows as

$$\left.\frac{dW(c,y;\alpha)}{d\varepsilon}\right|_{\varepsilon=0} = -\sum_{j=1}^{n} f_j G_j(\delta_j)\alpha_j + \left[1 - \sum_{j=1}^{n} f_j G_j(\delta_j)\right] \alpha_0 \left.\frac{d\tilde{c}_0}{d\varepsilon}\right|_{\varepsilon=0}$$
$$> -\sum_{j=1}^{n} f_j G_j(\delta_j)\alpha_j + \sum_{j=1}^{n} f_j G_j(\delta_j)\alpha_0 = \alpha_0 - \alpha_M \ ,$$

where $\alpha_M = 1$ is the average social weight in the population. For all $\alpha \in \mathcal{A}^{\chi}$, $\alpha_0 > \alpha_M$. Hence, an increase in ε is strictly welfare-increasing, and the initial allocation with $c_0 \leq 0$ is not welfare-maximizing.

(ii) For part *(a)*, there exists a number $k^\alpha \in (0,n]$ with the required properties by Lemma 19. Assume that $k^\alpha = n$, i.e., that labor supply in all skill groups is upwards distorted at the extensive margin. The feasibility condition then requires that $c_0 = \sum_{j=1}^n f_j G_j(\delta_j^\alpha)\left(y_j^\alpha - c_j^\alpha + c_0^\alpha\right) = \sum_{j=1}^n f_j G_j(\delta_j^\alpha)\left(y_j^\alpha - h(y_j^\alpha,\omega_j) - \delta_j^\alpha\right) < 0$. This is inconsistent with part (i) of this Lemma. Hence, $k^\alpha < n$ for all $\alpha \in \mathcal{A}^{\chi}$.

For the statement in part (b), note that labor supply in skill group j can only be upwards distorted at the intensive margin if $j-1$ and j are upwards-linked, i.e., $\nu_{j-1}^U > 0$ (see Lemma 16). By Lemma 17 (ii), this can only be true if labor supply in skill group j is upwards distorted at the extensive margin.

(iii) For all $\alpha \in \mathcal{A}^{\chi}$, $\delta_n^\alpha < y_n^\alpha - h(y_n^\alpha,\omega_j)$ as argued in the proof to part (ii). Hence, $n-1$ and n cannot be upwards-linked by Lemma 17. If $n-1$ and n are downwards-linked or unlinked, $y_n^\alpha = y_n^{\alpha R}$: Labor supply in skill group n is undistorted at the intensive margin. This moreover implies that $y_n^\alpha - h(y_n^\alpha,\omega_n) = \delta^*(n)$. Hence, $\delta_n^\alpha < \delta^*(\omega_n)$: labor supply in skill group is downwards distorted at the extensive margin.

□

To prove Proposition 1, it only remains to show that the optimal tax problem has a solution with $\delta_j \in \left[\underline{\delta},\bar{\delta}\right)$ for any $\alpha \in \mathcal{A}^{\chi}$.

Lemma 21. *For all $\alpha \in \mathcal{A}^{\chi}$, the optimal tax problem has a maximum (c^α, y^α) with $\delta_j \in \left[\underline{\delta},\bar{\delta}\right)$.*

Proof. Fix some $\alpha \in \mathcal{A}^{\chi}$. By Lemma 5, the FOCs with respect to y_j and δ_j are satisfied by a unique tuple $(y_j^\alpha, \delta_j^\alpha)$ if the local IC constraints between skill groups j, $j-1$ and $j+1$ are non-binding. In the following, I only consider the cases where skill group j is either downwards-linked or upwards-linked to skill group $j+1$ only. Similar proofs are available on request for cases where some skill groups k and $l > k+1$ are downwards-linked or upwards-linked.

First, assume that the downward IC (29) between skill groups j and $j+1$ is binding. For simplicity, I henceforth write $H_j(y_j) := h(y_j,\omega_j) - h(y_j,\omega_{j+1})$. The FOCs with respect to c_j, c_{j+1} and y_j (see Lemma 16) can be combined to get the optimality conditions

$$
\begin{aligned}
Z_1(y_j, \delta_j) = \quad & B_j(\delta_j, y_j) + B_{j+1}(\delta_{j+1}, y_{j+1}) & = 0 \\
Z_2(y_j, \delta_j) = \quad & B_{j+1}(\delta_{j+1}, y_{j+1}) + f_j G_j(\delta_j) C_j(y_j) & = 0 \, ,
\end{aligned}
$$

where $B_k(\delta_k, y_k) := f_k \left[G_k(\delta_k)(\alpha_k - 1) + g_k(\delta_k)\left(y_k - h(y_k, \omega_k) - \delta_k\right)\right]$ and $C_j(y_j) := [1 - h_y(y_j, \omega_j)] / H_j(y_j)$. By the binding downward IC constraint, we have $1 - h_y(y_j, \omega_j) > 0$, $C_j > 0$, $B_{j+1} < 0$ and $B_j > 0$ in allocation (c^α, y^α). Z_1 and Z_2 are continuous in y_j, δ_j and δ_{j+1}. Moreover, I will show that Z_1 has a root in δ_j at which $\partial Z_1 / \partial \delta_j < 0$, and that Z_2 has a root in y_j at which $\partial Z_2 / \partial y_j < 0$.

To start, consider function Z_1 for some fixed level $y_j < y_j^{\alpha R}$. Function $B_j(\delta_j, y_j^{\alpha R})$ has a unique root $\delta_{j1} \in \left(\underline{\delta}, \bar{\delta}\right)$ for any $\alpha_k \in [0, \chi)$ by Lemma 5. Function $\tilde{B}_{j+1}(\delta_j) := B_{j+1}(\delta_j + H_j(y_j^{\alpha R}), y_{j+1}^{\alpha R})$ either has a unique root δ_{j2} in $\left(\underline{\delta}, \bar{\delta}\right)$ as well, or it is negative for all $\delta_j \in \Delta$. In the first case, $\underline{\delta} < \delta_{j2} < \delta_{j1} < \bar{\delta}$. By Lemma 5, B_k is strictly decreasing in δ_k at its roots for any $k \in J$. For any $y_j < y_j^{\alpha R}$, the root δ_{j2} is increased, while the root δ_{j1} is decreased. As B_j and B_{j+1} are continuous, Z_1 must have a root in the interval $(\delta_{j2}, \delta_{j1})$. In the second case, if $\delta_j^\alpha = \underline{\delta}$ results if and only if $B_j(\underline{\delta}, y_j^{\alpha R}) + B_{j+1}(\underline{\delta} + H_j(y_j^{\alpha R}), y_{j+1}^{\alpha R}) < 0$.

Next, consider Z_2 for some fixed $\delta_j \in \left(\underline{\delta}, \delta_j^{\alpha R}\right)$. At any root of Z_2 in y_j, we have $B_{j+1} < 0$ and $C_j(y_j) > 0$. Note that $C_j'(y_j) = -\frac{h_y(y_j, \omega_j) H_j'(y_j) + (1 - h_y(y_j, \omega_j)) H_j''(y_j)}{H_j'(y_j)^2}$. By $h_{y\omega} < 0$ and $h_{yy\omega} \leq 0$, $C_j'(y_j) < 0$ for all $y_j \leq y_j^{\alpha R}$. The root y_{j1} of B_{j+1} must again be located left of $y_j^{\alpha R}$, the root of C_j. Hence, function Z_2 has a root y_j in the interval $\left(y_{j1}, y_j^{\alpha R}\right)$.

Second, assume that the upward IC constraint (33) is binding. Recall that this only results for $\alpha_j 1$ and $\alpha_{j+1} > 1$. Then, the optimal tuple (δ_j, y_{j+1}) is implicitly defined by

$$
\begin{aligned}
Z_3(y_{j+1}, \delta_j) = B_j(\delta_j, y_j) + B_{j+1}(\delta_{j+1}, y_{j+1}) & = 0 \\
Z_4(y_{j+1}, \delta_j) = B_{j+1}(\delta_{j+1}, y_{j+1}) + f_{j+1} G_{j+1}(\delta_{j+1}) C_{j+1}(y_{j+1}) & = 0 \, ,
\end{aligned}
$$

where $C_{j+1}(y_{j+1}) := [1 - h_y(y_{j+1}, \omega_{j+1})] / H_j(y_{j+1})$. By the binding upward IC constraint, we have $1 - h_y(y_{j+1}, \omega_{j+1}) < 0$, $C_{j+1}(y_{j+1}) < 0$, $B_{j+1} > 0$ and $B_j < 0$ in this optimum.

Again, I start by showing that Z_3 has a root in δ_j for some fixed $y_{j+1} > y_{j+1}^{\alpha R}$. Function B_j has a unique root $\delta_{j3} \in \left(\delta^*(\omega_j), \lim_{\alpha_j \to \chi} \delta_j^{\alpha R}\right)$. For any $y_{j+1} < y_{j+1}^{\alpha R}$, function $\tilde{B}_{j+1}(\delta_j) = B_{j+1}(\delta_j + H_j(y_{j+1}), y_{j+1})$ has a unique root δ_{j4} such that $\delta_{j4} + H_j(y_{j+1})$ is below the optimal $\delta_{j+1}^{\alpha R}$ for $\alpha_{j+1} < \chi$. Both functions are strictly positive left of these roots, and strictly negative right of these roots. As $B_{j+1} > 0$ and $B_j < 0$ must be satisfied, $\delta_{j3} < \delta_{j4}$. Hence, Z_3 must have a root in δ_j in the interval $(\delta_{j3}, \delta_{j4})$.

Finally, consider function Z_4 for some fixed $\delta_j > \delta_j^\alpha$. B_{j+1} has a unique root in y_{j+1} above $y_{j+1}^{\alpha R}$, while $C_{j+1}(y_{j+1})$ has a unique root at $y_{j+1}^{\alpha R}$. Both B_{j+1} and C_{j+1} are strictly decreasing at their roots. Hence, Z_4 has a root in y_{j+1} with $y_{j+1} > y_{j+1}^{\alpha R}$. □

Proof of Proposition 2

I prove Proposition 2 in two steps, starting with a preliminary Lemma.

Lemma 22. *Consider a partially relaxed version of the optimal tax problem in which all constraint from the relaxed problem and, additionally, all local IC constraints between the workers in skill groups 1 and k are taken into account. For any $\alpha \in \mathcal{A}_k^U$, optimal output in the solution $(c^{\alpha P}, y^{\alpha P})$ to this partially relaxed problem is*

- *upwards distorted at the intensive margin in skill groups $\{2, \dots, k\}$;*
- *upwards distorted at the extensive margin in skill groups $\{1, \dots, k\}$.*

Proof. Fix some $\alpha \in \mathcal{A}_k^U$. By Lemma 15, the relaxed problem's solution $(c^{\alpha R}, y^{\alpha R})$ violates the upward IC for at least one pair of skill groups $(j, j+1)$ with $j \in \{1, \dots, k-1\}$, where $\alpha_{j+1} < \beta_j^D(\alpha_j)$.

For the first step, consider the intermediate problem A that takes into account the local IC constraints between skill groups j and $j+1$, but ignores the ICs between all other skill pairs. I denote the solution to this problem by (c^A, y^A). By $\alpha_{j+1} > \beta_j^U(\alpha_j)$, the upward IC is binding with $\nu_j^U > 0$. From Lemma 16, we know that $y_{j+1}^A > y_{j+1}^{\alpha R}$, $\delta_j^A > \delta_j^{\alpha R}$ and $\delta_{j+1}^A < \delta_{j+1}^{\alpha R}$. For any other skill groups l, we have $y_l^A = y_l^{\alpha R}$ and $\delta_l^A = \delta_l^{\alpha R}$. In particular, this is true for skill group $j+2$. If $j+2 \leq k$, (c^A, y^A) violates the upward IC constraint between skill groups $j+1$ and $j+2$, because $\delta_{j+2}^{\alpha R} - \delta_{j+1}^{\alpha R} \geq h(y_{j+2}^{\alpha R}, \omega_{j+1}) - h(y_{j+2}^{\alpha R}, \omega_{j+2})$ by construction for any $\alpha \in \mathcal{A}_k^U$.

For the second step, consider the intermediate problem $A2$ that takes into account the local IC constraints between the skill groups j, $j+1$ and $j+2$. In the solution (c^{A2}, y^{A2}) to problem $A2$, both upward IC constraints are binding with $\nu_j^U > 0$ and $\nu_{j+1}^U > 0$. Consequently, we have $y_{j+2}^{A2} > y_{j+2}^{\alpha R}$, $y_{j+1}^{A2} > y_{j+1}^{\alpha R}$, $\delta_{j+2}^{A2} < \delta_{j+2}^{\alpha R}$ and $\delta_j^A > \delta_j^{\alpha R}$, while $y_{j+3}^{A2} = y_{j+3}^{\alpha R}$ and $\delta_{j+3}^{A2} = \delta_{j+3}^{\alpha R}$. These inequalities imply that, if $j+3 \leq k$, (c^{A2}, y^{A2}) violates the upward IC between skill groups $j+2$ and $j+3$ for any $\alpha \in \mathcal{A}_k^U$. The same arguments can be repeated to show that, in the solution to problem B that takes into account the local ICs between skill groups $\{j, \dots, k\}$ only, all upward IC constraints are binding and labor supply in each skill group $j \in \{j+1, \dots, k\}$ is upwards distorted at the intensive margin.

For the third step, note that the solution to problem B involves $y_j^B = y_j^{\alpha R}$, $\delta_j^B > \delta_j^{\alpha R}$ and $\delta_{j-1}^B = \delta_{j-1}^{\alpha R}$. Hence, this allocation violates the upward IC constraint between skill groups $j-1$ and j for any $\alpha \in \mathcal{A}_k^U$. In problem $B2$ that takes into account the local ICs between skill groups $\{j-1, \dots, k\}$ only, all upwards IC constraints are binding again. Hence, we have $y_j^{B2} > y_j^{\alpha R}$, $y_{j-1}^{B2} = y_{j-1}^{\alpha R}$, $\delta_{j-1}^{B2} > \delta_{j-1}^{\alpha R}$ and $\delta_{j-2}^{B2} = \delta_{j-2}^{\alpha R}$, which ensures that the upward IC between skill groups $j-2$ and $j-1$ is violated. The same arguments can be repeated to show that, in allocation $(c^{\alpha P}, y^{\alpha P})$, all upward IC constraints along the skill dimension are binding and that labor supply is downwards distorted at the intensive margin in each skill group $j \in \{2, \dots, k\}$.

Finally, Lemma 17 implies that, if skill groups 1 and k are upwards-linked, then labor supply in all these skill groups is upwards distorted at the extensive margin. □

Building on Lemma 22, I can now prove Proposition 2.

Proof. Fix some $\alpha \in \mathcal{A}_k^U$ such that $\alpha_{j+1} \leq \beta_j^U(\alpha_j)$ for all $j \in \{k, \ldots, n-1\}$, i.e., that the solution to the relaxed problem violates no upward IC except the ones between skill groups 1 and k. This also implies that $\alpha_j > \alpha_{j+1} > 1$ for all $j \in \{1, \ldots, k-1\}$. The following proof focuses on this case. The results derived below hold for the alternative case that $(c^{\alpha R}, y^{\alpha R})$ violates additional upward ICs among higher-skilled workers *a forteriori*. To economize on notation, I write $H_k(y_j) \coloneqq h(y_j, \omega_k) - h(y_j, \omega_{k+1})$ in the following.

Consider an auxiliary problem A that takes into account all local IC constraints except the ones between skill groups k and $k+1$. By Lemma 22, the solution (c^A, y^A) involves binding upward IC between skill groups j and $j+1$ with $\nu_j^U > 0$ for all $j \in \{1, \ldots, k-1\}$, while all other local IC constraints are non-binding with $\nu_j^D = \nu_j^U = 0$ for all $j \in \{k+1, \ldots, n-1\}$. Recall that the relaxed problem's solution satisfies both neglected IC constraints for any $\alpha \in \mathcal{A}_k^U$, i.e.,

$$H_k\left(y_k^{\alpha R}\right) \leq \delta_{k+1}^{\alpha R} - \delta_k^{\alpha R} \leq H_k\left(y_{k+1}^{\alpha R}\right) \ .$$

By Lemma 16, the solution (c^A, y^A) involves $y_k^A > y_k^{\alpha R}$, $\delta_k^A < \delta_k^{\alpha R}$ and $y_j^A = y_j^{\alpha R}$ as well as $\delta_j^A = \delta_j^{\alpha R}$ for all $j > k$. By $h_{yw} < 0$, $y_k^A > y_k^{\alpha R}$ implies that $H_k(y_k^A) > H_k(y_k^{\alpha R})$. Hence, (c^A, y^A) may violate the downward IC between k and $k+1$, the corresponding upward IC or none of them. If (c^A, y^A) satisfies both IC constraints, we have $(c^\alpha, y^\alpha) = (c^A, y^A)$. In the following, I consider the remaining two cases.

First, assume that (c^A, y^A) violates the downward IC constraint between skill groups k and $k+1$. Consider problem B that takes into account all local IC constraints except those between $k+1$ and $k+2$. In its solution (c^B, y^B), the upward IC between $k-1$ and k and the downward IC between k and $k+2$ are binding with $\nu_{k-1}^U > 0$ and $\nu_k^D > 0$. Hence, we have $\delta_k^B < \delta_k^{\alpha R}$ and $\delta_{k+1}^B > \delta_{k+1}^{\alpha R}$. By the binding downward IC, we have $H_k(y_k^B) = \delta_{k+1}^B - \delta_k^B > \delta_{k+1}^{\alpha R} - \delta_k^{\alpha R} \geq H_k(y_k^{\alpha R})$. This ensures that $y_k^B > y_k^{\alpha R}$ by $h_{y\omega} < 0$. Note further that (i) $\delta_k^B < \delta_k^A$ and (ii) $y_j^B = y_j^{\alpha R}$ and $\delta_j^B = \delta_j^{\alpha R}$ for all $j > k+1$. By (i), the upward IC between $k-1$ and k continues to be binding. By (ii), allocation (c^B, y^B) satisfies the (neglected) upward IC between skill groups $k+1$ and $k+2$, and may satisfy or violate the corresponding downward IC. Adding the IC constraints for all skill groups above $k+1$ stepwise, similar arguments as in the proof to Proposition 5 in Appendix B.1 can be applied to show that the optimal allocation (c^α, y^α) satisfies the following conditions:

(a) the upward ICs between all skill levels 1 and k are binding and $y_j^\alpha > y_j^{\alpha R}$ for all $j \in \{2, \ldots, k\}$,

(b) y^{α} is upwards distorted at the extensive margin in all skill groups 1 to k by Lemma 17,

(c) there is a unique skill group $l \in \{k+1, \ldots, n\}$ such that the skill groups k and l are downwards-linked, while all skill groups between l and n are unlinked.

Second, assume that (c^A, y^A) violates the upward IC constraint between skill groups k and $k+1$. Then, allocation (c^B, y^B) involves binding upward ICs between $k-1$ and k as well as between k and $k+1$. Hence, we have $\delta^B_{k+1} < \delta^{\alpha R}_{k+1}$ and $y^B_{k+1} > y^{\alpha R}_{k+1}$. By the previous arguments, allocation (c^B, y^B) may violate the downward IC between $k+1$ and $k+2$, the corresponding upward IC or none of both. In all three cases, skill groups 1 and $k+1$ continue to be upwards-linked in (c^{α}, y^{α}) and $y^{\alpha}_j > y^{\alpha R}_j$ for all $j \in \{2, \ldots, k+1\}$ as shown above. Besides, labor supply in all skill groups $j \in \{1, \ldots, k+1\}$ is upwards distorted at the extensive margin in (c^{α}, y^{α}) by Lemma 17. The same arguments can be repeated to show that there always exists a skill group $l \in \{k+1, \ldots, n-1\}$ such that the skill groups 1 and l are upwards-linked and (a) either all skill groups between l and n are unlinked or (b) there is a skill group $q \in \{l, \ldots, n\}$ such that the skill groups l and q are downwards-linked, while all skill groups between q and n are unlinked. Finally, note that skill groups 1 and n cannot be upwards-linked, i.e., $l \neq n$, by Proposition 1. □

Proof of Proposition 3

Proposition 3 provides sufficient conditions for the existence of *regular* combinations of Ψ and γ such that the endogenous (average) welfare weights $\bar{\alpha}$ are elements of the set $\mathcal{A}^U$. It is proven in Lemmas 23 and 24 below by example, for Ψ being equal to the identity function, i.e., $\Psi(x) = x$ for all $x \in \mathbb{R}$.

Lemma 23. *Fix some $k \in \{2, \ldots, n-1\}$ and consider a sequence $\alpha'(\tau) = \left(\alpha'_j(\tau)\right)_{j=k}^{n}$ such that $\alpha'_k(\tau) = \tau$ and $\alpha'_{j+1}(\tau) = \beta^D_j(\alpha'_j(\tau))$ for all $j \in \{k, \ldots, n-1\}$. For any $\tau \in (1, \chi)$, there is a unique natural number $m^k \geq k+1$ such that $\alpha'_j(\tau) < 1$ if and only if $j \in \{m^k, \ldots, n\}$.*

Proof. By Lemma 14, $\beta^D_j(\alpha_j) < \alpha_j$ for all $\alpha_j \geq 1$ and $\beta^D_j(\alpha_j) < 1$ for all $\alpha_j < 1$ (for any $j \in J_{-n}$). Hence, there exists at most one natural number m such that $\alpha'_{m-1} \geq 1$ and $\alpha'_m < 1$. It remains to show that there is a real number $\mu' > 0$ such that $\alpha_j - \beta^D_j(\alpha_j) > \mu'$ for all $j \in J$ and all $\alpha_j \in [1, \chi)$. If this is true, $\alpha'(\tau) < \max\{\tau - (j-k)\mu', 1\}$ for all $j > k$. Hence, $\alpha'_j(\tau) < 1$ if and only if $j > k + \frac{\tau - 1}{\mu'}$, which directly implies that $m^k < k + 1 + \frac{\tau-1}{\mu'}$ and that $\alpha_n < 1$ whenever n is large enough. The following proof shows that this is true under Condition 3.

With some abuse of notation, I henceforth denote by $\delta^{\alpha R}_k(\alpha_k)$ the level of δ_k in $(c^{\alpha R}, y^{\alpha R})$ given social weight α_k. By construction, function β^D_j satisfies

$$\delta^{\alpha R}_{j+1}\left(\beta^D_j(x)\right) - \delta^{\alpha R}_j(x) = h\left(y^{\alpha R}_j, \omega_j\right) - h\left(y^{\alpha R}_j, \omega_{j+1}\right) \ ,$$

while, for all $x \geq 1$, $\delta^{\alpha R}_{j+1}(x) - \delta^{\alpha R}_{j}(x) \geq \delta^*(\omega_{j+1}) - \delta^*(\omega_j)$ by Lemma 11. Consequently, we have

$$\begin{aligned}
\delta^{\alpha R}_{j+1}(x) - \delta^{\alpha R}_{j+1}(\beta^D_j(x)) &\geq \delta^*(\omega_{j+1}) - \delta^*(\omega_j) - \left[h\left(y^{\alpha R}_j, \omega_j\right) - h\left(y^{\alpha R}_j, \omega_{j+1}\right)\right] \\
&= y^{\alpha R}_{j+1} - h\left(y^{\alpha R}_{j+1}, \omega_{j+1}\right) - \left[y^{\alpha R}_j - h\left(y^{\alpha R}_j, \omega_{j+1}\right)\right] \\
&= \int_{y^{\alpha R}_j}^{y^{\alpha R}_{j+1}} \left[1 - h_y(y, \omega_{j+1})\right] dy \\
&= \int_{y^{\alpha R}_j}^{y^{\alpha R}_{j+1}} \left[1 - \frac{1}{1 + \frac{h_y(y^{\alpha R}_{j+1}, \omega_{j+1}) - h_y(y, \omega_{j+1})}{h_y(y, \omega_{j+1})}}\right] dy,
\end{aligned}$$

where I exploit that $h_y(y^{\alpha R}_{j+1}, \omega_{j+1}) = 1$. Let $\hat{y}(\omega) := \arg\max_{y \in \mathbb{R}} y - h(y, \omega)$, and $\hat{w}(y) := (\hat{y})^{-1}(y)$ the corresponding inverse function. Then, the following inequality holds

$$\begin{aligned}
\frac{h_y(y^{\alpha R}_{j+1}, \omega_{j+1}) - h_y(y, \omega_{j+1})}{h_y(y, \omega_{j+1})} &= \int_{\hat{w}(y)}^{\omega_{j+1}} \frac{h_{yy}(\hat{y}(\omega), \omega_{j+1})}{h_y(y, \omega_{j+1})} \frac{d\hat{y}(\omega)}{d\omega} d\omega \\
&= -\int_{\hat{w}(y)}^{\omega_{j+1}} \frac{h_{yy}(\hat{y}(\omega), \omega_{j+1})}{h_y(y, \omega_{j+1})} \frac{h_{y\omega}(\hat{y}(\omega), \omega)}{h_{yy}(\hat{y}(\omega), \omega)} d\omega \\
&> -\int_{\hat{w}(y)}^{\omega_{j+1}} \frac{\hat{y}(\omega) h_{yy}(\hat{y}(\omega), \omega_{j+1})}{h_y(\hat{y}(\omega), \omega_{j+1})} \frac{\omega h_{y\omega}(\hat{y}(\omega), \omega)}{\hat{y}(\omega) h_{yy}(\hat{y}(\omega), \omega)} \frac{1}{\omega} d\omega \\
&\geq \int_{\hat{w}(y)}^{\omega_{j+1}} \frac{\mu_2}{\mu_1} \frac{1}{\omega} d\omega = \frac{\mu_2}{\mu_1} \ln\left(\frac{\omega_{j+1}}{\hat{w}(y)}\right),
\end{aligned}$$

where the last line inequality follows from Condition 3. Hence, we can derive the following lower bound

$$\begin{aligned}
\delta^{\alpha R}_{j+1}(x) - \delta^{\alpha R}_{j+1}(\beta^D_j(x)) &> \int_{y^{\alpha R}_j}^{y^{\alpha R}_{j+1}} \left[1 - \frac{1}{1 + \frac{\mu_2}{\mu_1} \ln\left(\frac{\omega_{j+1}}{\hat{w}(y)}\right)}\right] dy = \int_{y^{\alpha R}_j}^{y^{\alpha R}_{j+1}} \frac{\ln\left(\frac{\omega_{j+1}}{\hat{w}(y)}\right)}{\frac{\mu_1}{\mu_2} + \ln\left(\frac{\omega_{j+1}}{\hat{w}(y)}\right)} dy \\
&> \int_{y^{\alpha R}_j}^{\hat{y}(\omega')} \frac{\ln\left(\frac{\omega_{j+1}}{\omega'}\right)}{\frac{\mu_1}{\mu_2} + \ln\left(\frac{\omega_{j+1}}{\omega'}\right)} dy = \frac{\ln\left(\frac{\omega_{j+1}}{\omega'}\right)}{\frac{\mu_1}{\mu_2} + \ln\left(\frac{\omega_{j+1}}{\omega'}\right)} \left[\hat{y}(\omega') - y^{\alpha R}_j\right] \\
&\geq \frac{\ln\left(\frac{\omega_{j+1}}{\omega'}\right)}{\frac{\mu_1}{\mu_2} + \ln\left(\frac{\omega_{j+1}}{\omega'}\right)} \mu_2 \ln\left(\frac{\omega'}{\omega_j}\right) y^{\alpha R}_j
\end{aligned}$$

for any $\omega' \in (\omega_j, \omega_{j+1})$. In particular, let $\omega' = \sqrt{\omega_j \omega_{j+1}}$ and recall that, by assumption, $\omega_{j+1}/\omega_j \geq 1 + \varepsilon$ for all $j \in J_{-n}$ and some number $\varepsilon > 0$. Denoting $\tilde{\varepsilon} = \ln(1 + \varepsilon)$, we get

$$\delta^{\alpha R}_{j+1}(x) - \delta^{\alpha R}_{j+1}(\beta^D_j(x)) > \underbrace{\frac{(\mu_2 \tilde{\varepsilon})^2}{\mu_1 + \mu_2 \tilde{\varepsilon}}}_{:= \mu_3} y^{\alpha R}_j,$$

where μ_3 is bound away from zero for any $j \in J_{-n}$.

From equation (32), the left-hand side of the last inequality can also be written as

$$\int_{\beta_j^D(x)}^{x} \frac{d\delta_{j+1}^{\alpha R}(x')}{d\alpha} dx' = \int_{\beta_j^D(x)}^{x} \frac{\left[A_{j+1}\left(\delta_j^{\alpha R}(x')\right)\right]^{-1}}{1+(x'-1)\left[1-\frac{a_j\left(\delta_j^{\alpha R}(x')\right)}{A_j\left(\delta_j^{\alpha R}(x')\right)}\right]} dx'$$

By Lemma 2, $A_j(\delta_j^{\alpha R}) > A_n(\delta_n^{\alpha R})$ in any allocation that satisfies all downward IC constraints. Moreover, $\alpha_j'(\tau) \leq \tau$ for all $j \geq k$, and $1 - a_j\left(\delta_j^{\alpha R}\right)/A_j\left(\delta_j^{\alpha R}\right) < 1/(\chi - 1)$ for any $\alpha \in \mathcal{A}^{\chi}$ by the construction of χ (see Lemma 5). Hence, $d\delta_j^{\alpha R}(x')/d\alpha$ is strictly smaller than $\mu_4 := A_n[\delta_n^{\alpha R}(\tau)]^{-1}(\chi-1)/(\chi-\tau)$ for all $j \in J$ and $x' \in [1, \tau]$. For all $j \geq k$ and $\alpha \in [1, \tau]$, we consequently have

$$\int_{\beta_j^D(x)}^{x} \mu_4 dx' = \mu_4\left[x - \beta_j^D(x)\right] > \mu_3 y_j^{\alpha R}$$
$$\Leftrightarrow x - \beta_j^D(x) > \frac{\mu_3}{\mu_4} y_k^{\alpha R} =: \mu' \, .$$

As argued above, this ensures that $\tau - \alpha_j'(\tau) = \sum_{l=k}^{j-1}\left[\alpha_k'(\tau) - \beta_k^D\left(\alpha_k'(\tau)\right)\right] > (j-k)\mu'$ if $\alpha'(\tau) \geq 1$, and $\alpha_j'(\tau) < 1$ for all $j > k + \frac{\tau-1}{\mu'}$. Hence, there exists a level $m^k \in [k+1, k+(\tau-1)/\mu')$ such that, if $n > m^k$, we have $\alpha_j < 1$ if and only if $j \geq m^k$. □

Lemma 24. *Define* $\hat{a}_U^k(\zeta) := \min\left\{a_1^u(\zeta), \ldots, a_{k-1}^u(\zeta)\right\}$*. Let Condition 3 and* $\frac{\omega_{j+1}}{\omega_j} < \hat{a}_U^k(\zeta)$ *be satisfied for all* $j \in \{1, \ldots, k-1\}$ *and some* $\zeta \in (1, \chi)$*. Then, there exist a number* $m^k \geq k+1$ *and two vectors* $(\phi_j^k)_{j=1}^n$*,* $(\delta_j^k)_{j=1}^n$ *with* $\phi_{j+1}^k \geq \phi_j^k$*,* $\phi_{m^k}^k > 1 \geq \phi_{m^k-1}^k$ *and* $\delta_j^k \in \left(\underline{\delta}, \bar{\delta}\right)$ *for all* $j \in J$ *such that, if*

(i) $n \geq m^k$ *and*

(ii) $\sum_{j=1}^n f_j G_j(\delta_j^k)\phi_j^k > 1$*,*

there exist regular combinations of Ψ *and* γ *for which* $\bar{\alpha} \in \mathcal{A}_k^U$*.*

Proof. Assume that Ψ is the identity function and fix some number $\hat{\delta} \in \left(\delta^*(\omega_n), \bar{\delta}\right)$. Recall that $\bar{\beta}_j$ is the fixed point of function β_j^U, which exists if $\omega_{j+1}/\omega_j < a_j^U(\zeta)$ for some $\zeta \in (1, \chi)$ under Conditions 1 and 2. Define $\hat{\beta}_j^k := \max\left\{\bar{\beta}_j, \ldots, \bar{\beta}_{k-1}\right\}$, $\gamma_1^k = \hat{\beta}_1^k$, $\gamma_j^k = \min\left\{\hat{\beta}_j^k, \beta_{j-1}^U(\gamma_{j-1})\right\}$ for all $j \in \{2, \ldots, k\}$ and $\gamma_j^k = \max\left\{\beta_{j-1}^D(\gamma_{j-1}), \underline{\beta}_j, \ldots, \underline{\beta}_{n-1}\right\}$ for all $j \in \{k+1, \ldots, n\}$. By construction, $\gamma_{j+1}^k \leq \gamma_j^k$ for all $j \in J_{-n}$. By Lemma 23, there is a number $m^k \geq k+1$ such that $\gamma_j < 1$ if and only if $j \in \left\{m^k, \ldots, n\right\}$ whenever $n \geq m^k$ (i.e., if (i) in the Lemma above holds).

Now, consider the type-dependent weighting function $\tilde{\gamma}^k : \Omega \times \Delta \to \mathbb{R}$ such that

$$\tilde{\gamma}^k(\omega_j, \delta) = \begin{cases} \gamma_j^k & \text{for } \ j \in J, \ \delta < \hat{\delta}\,, \\ \gamma_0^k = \frac{1-\sum_{j=1}^n f_j G_j(\hat{\delta})\gamma_j}{1-\sum_{j=1}^n f_j G_j(\hat{\delta})} & \text{for } \ j \in J, \ \delta \geq \hat{\delta}\,. \end{cases}$$

Let δ_j^k be equal to the level of δ_j in the relaxed problem's solution $\left(c^{\alpha R}, y^{\alpha R}\right)$ given $\alpha_j = \gamma_j$ for each $j \in J$. Then, the implied average weight $\bar{\alpha}_n$ (among workers in skill group n) equals $\gamma_n < 1$, and $\delta_n^k < \delta^*(\omega_n) < \hat{\delta}$. Moreover, the average weight of workers in skill group j is given by $\bar{\alpha}_j = \gamma_j^k$ and $\delta_j^k < \delta_n^k < \hat{\delta}$ for all $j \in J_{-n}$ (as all downward IC constraints are satisfied, see proof of Lemma 2). Finally, the average social weight of the unemployed is given by $\bar{\alpha}_0 = \left[1 - \sum_{j=1}^n f_j G_j\left(\delta_j^k\right)\gamma_j^k\right] / \left[1 - \sum_{j=1}^n f_j G_j\left(\delta_j^k\right)\right]$.

By construction, $\bar{\alpha}$ satisfies $\bar{\alpha}_{j+1} \geq \beta_j^U(\bar{\alpha}_j)$ for all $j \in \{1, \ldots, k-1\}$ and $\bar{\alpha}_{j+1} \geq \beta_j^D(\bar{\alpha}_j)$ for all $j \in \{k, \ldots, n-1\}$. Moreover, $\bar{\alpha}_j \geq \bar{\alpha}_{j+1}$ for all $j \in J$ and $\bar{\alpha}_j > \bar{\alpha}_{j+1}$ for all $j \in \left\{k, \ldots, m^k\right\}$. We also have $\bar{\alpha}_0 > \bar{\alpha}_1 = \hat{\beta}_1^k$ if and only if

$$\sum_{j=1}^n f_j G_j\left(\delta_j^k\right)\left[\hat{\beta}_1^k - \gamma_j^k\right] > \hat{\beta}_1^k - 1 \,. \tag{41}$$

Let $\phi_j^k = (\hat{\beta}_1^k - \gamma_j^k)/(\hat{\beta}_1^k - 1)$ for all $j \in J$. Then, condition (ii) in Lemma 24 ensures $\bar{\alpha}_0 > \hat{\beta}_1$.

For the final step, note that the average welfare weights $\bar{\alpha}$ are not an element of $\mathcal{A}^U$ given γ because $\bar{\alpha}_j = \bar{\alpha}_{j+1} = \bar{\beta}_j = \beta_j^U(\bar{\beta}_j)$ for at least one $j \in \{1, \ldots, k-1\}$. Consider the weighting function γ^ε such that

$$\gamma^\varepsilon(\omega_j, \delta) = \begin{cases} \gamma_j^\varepsilon = \gamma_j^k + \varepsilon_k & \text{for} \quad j \in \{1, \ldots, k\}, \delta < \hat{\delta}\,, \\ \gamma_j^\varepsilon = \max\left\{\beta_{j-1}^D\left(\gamma_{j-1}^\varepsilon\right), \underline{\beta}_j, \ldots, \underline{\beta}_{n-1}\right\} & \text{for} \quad j \in \{3, \ldots, n\}, \delta < \hat{\delta}\,, \\ \gamma_0^\varepsilon = \frac{1 - \sum_{j=1}^n f_j G_j\left(\delta_j^\varepsilon\right)\gamma_j^\varepsilon}{1 - \sum_{j=1}^n f_j G_j\left(\delta_j^\varepsilon\right)} & \text{for} \qquad j \in J\,, \delta \geq \hat{\delta}\,, \end{cases}$$

where δ_j^ε is the level of δ_j in the relaxed problem's solution given $\alpha_j = \gamma_j^\varepsilon$ for each $j \in J$. For each $j \in \{1, \ldots, k\}$, fix ε_j at some level in $\left(0, \chi - \hat{\beta}_1^k\right)$ such that $\varepsilon_j > \varepsilon_{j+1}$. By Lemma 13, for any j such that $\gamma_j^k = \gamma_{j+1}^k$, there is a unique number $\bar{\varepsilon}(\varepsilon_{j+1}) > 0$ such that $\gamma_{j+1}^\varepsilon \geq \beta_j^U(\gamma_j^\varepsilon)$ if and only if $\varepsilon_j - \varepsilon_{j+1} \in (0, \bar{\varepsilon}(\varepsilon_{j+1}))$. If the difference $\varepsilon_j - \varepsilon_{j+1}$ is sufficiently small for each $j \in \{1, \ldots, k-1\}$, the resulting average weight $\bar{\alpha}_0$ of the unemployed agents continues to be strictly larger than γ_1^ε. Hence, the implied average weights satisfy $\bar{\alpha}_j > \bar{\alpha}_{j+1}$ for all $j \in \{0, \ldots, m-1\}$. Consequently, $\bar{\alpha} \in \mathcal{A}^U$ results if $\gamma_{j+1}^\varepsilon < \gamma_j^\varepsilon$ for all $j \in \{m, \ldots, n-1\}$ as well. (If γ^ε involves $\gamma_{j+1}^\varepsilon = \gamma_j^\varepsilon$ for some $j \geq m$, then one can construct a weighting function γ that is strictly decreasing over the skill dimension and has otherwise identical properties, i.e., for which $\bar{\alpha} \in \mathcal{A}_k^U$.) □

Proof of Lemma 7

Proof. First, note that minimizing the deadweight loss (5) is equivalent to maximizing the term $\sum_{j=1}^n f_j \int_{\underline{\delta}}^{\delta_j} g_j(\delta)\left[y_j - h(y_j, \omega_j) - \delta\right] d\delta$ over c and y. The Lagrangian for the problem of efficient redistribution is hence given by

$$\begin{aligned}\mathcal{L} &= \sum_{j=1}^{n} f_j \int_{\underline{\delta}}^{\delta_j} g_j(\delta)\left[y_j - h(y_j,\omega_j) - \delta\right] d\delta + \lambda_F \left[\sum_{j=3}^{n} f_j G_j(\delta_j)(y_j - c_j) - R\right] \\ &+\lambda_E \left[\sum_{j=1}^{2} f_j G_j(\delta_j)(c_j - y_j) + \sum_{j=1}^{n} f_j \left[1 - G_j(\delta_j)\right] c_0 - R\right] \\ &+\mu\left[c_1 - h(y_1,\omega_1) - c_2 + h(y_2,\omega_1)\right] ,\end{aligned}$$

where I have included the constraints that resources R are transferred away from the workers in skill groups 3 and higher (Lagrange parameter λ_F), the same resources are transferred towards the unemployed and the workers in skill groups 1 and 2 (λ_E), and the upward IC constraint between the workers in skill groups 1 and 2 (μ). Note that the first two constraints are binding in the solution of the efficient redistribution problem for any $R > 0$. Hence, λ_E and λ_F are strictly positive in any solution.

First, I solve the problem ignoring the upward IC constraint, and denote the solution by (c^{ER}, y^{ER}). The FOCs with respect to c_j and y_j for $j \in \{1,2\}$ are given by

$$\begin{aligned}\mathcal{L}_{c_j} &= (1-\lambda_E) f_j g_j(\delta_j^{ER})\left[y_j^{ER} - h(y_j^{ER},\omega_j) - \delta_j^{ER}\right] + \lambda_E f_j G_j(\delta_j^{ER}) \overset{!}{=} 0 \quad \text{, and} \\ \mathcal{L}_{y_j} &= f_j G_j(\delta_j^{ER})\left[1 - h_y(y_j^{ER},\omega_j)\right] \\ &\quad -h_y(y_j^{ER},\omega_j)(1-\lambda_E) f_j g_j(\delta_j^{ER})\left[y_j^{ER} - h(y_j^{ER},\omega_j) - \delta_j^{ER}\right] - \lambda_E f_j G_j(\delta_j^{ER}) \overset{!}{=} 0 .\end{aligned}$$

The combination of both FOCs yields that labor supply in both low-skill groups is undistorted at the intensive margin, $h_y(y_j^{ER},\omega_j) = 1$, which also implies $y_j^{ER} - h(y_j^{ER},\omega_j) = \delta^*(\omega_j)$. Rearranging the first condition, we additionally get the inverse elasticity rule (24). From the second-order condition, λ_E has to attain a value below some bound $\hat{\lambda} < 1$ at any solution (the exact level of $\hat{\lambda}$ depends on the type distribution K). Hence, $\delta_j^{ER} > \delta^*(\omega_j)$ for $j \in \{1,2\}$ and any $R > 0$ such that an interior solution exists. From Lemma 2, we know that $\eta_1(c,y) = \frac{g_1(\delta_1)}{G_1(\delta_1)} > \eta_2(c,y) = \frac{g_2(\delta_2)}{G_2(\delta_2)}$ in any incentive-compatible allocation, which implies that $c_2^{ER} - y_2^{ER} > c_1^{ER} - y_1^{ER}$. Hence, allocation (c^{ER}, y^{ER}) satisfies the downward IC (22), and might satisfy or violate the upward IC (23).

Fix a number $R > 0$, the type distribution K and all skill levels except ω_2. Similar arguments as in the proof of Lemma 13 (i) can be applied to show that there is some number $a^E > 1$ such that the upward IC constraint is violated by the allocation (c^{ER}, y^{ER}) whenever $\omega_2 \in \left(\omega_1, a^E\omega_1\right)$. First, consider the limit case $\omega_2 = \omega_1$. In this case, $\lambda_E \in (0,1)$ and $\delta_2^{ER} \geq \delta_1^{ER}$ by Condition 1. As shown in the proof of Lemma 13 (i), allocation (c^E, y^E) unambiguously violates the upward IC constraint after a marginal increase in ω_2 for any fixed $\lambda_E \in (0,1)$. Because ω_2 and all other variables enter the maximization program continuously, ω_2 also affects the value of the Lagrange parameter λ_E continuously. Hence, λ_E is bounded away from 0 and 1 after a marginal increase in ω_2. Thus, there is a number

$a^E > 1$ such that the upward IC constraint is violated for any $\omega_2 \in \big(\omega_1, a^E\omega_1\big)$.

Next, consider a combination of $R > 0$ and $\omega_2 \in (\omega_1, \omega_3)$ such that (c^{ER}, y^{ER}) violates the upward IC constraint. Taking into account this constraint for the Lagrangian, the adjusted FOCs with respect to c_1, c_2 and y_1 are given by

$$\begin{aligned}
\mathcal{L}_{c_1} &= (1-\lambda_E) f_1 g_1(\delta_1^E)\left[y_1^E - h(y_1^E,\omega_1) - \delta_1^E\right] + \lambda_E f_1 G_1(\delta_1^E) + \mu \overset{!}{=} 0 \ , \\
\mathcal{L}_{c_2} &= (1-\lambda_E) f_2 g_2(\delta_2^E)\left[y_2^E - h(y_2^E,\omega_2) - \delta_2^E\right] + \lambda_E f_2 G_2(\delta_2^E) - \mu \overset{!}{=} 0 \ \text{, and} \\
\mathcal{L}_{y_2} &= f_2 G_2(\delta_2^E)\left[1 - h_y(y_2^E,\omega_2)\right] - h_y(y_2^E,\omega_2)(1-\lambda_E) f_2 g_2(\delta_2^E)\left[y_2^E - h(y_2^E,\omega_2) - \delta_2^E\right] \\
&\quad -\lambda_E f_2 G_2(\delta_2^E) + \mu h_y(_2,\omega_1) \overset{!}{=} 0 \ .
\end{aligned}$$

Combining the two latter conditions shows that, as usual, y_2^E is upwards distorted if and only if the upward IC constraint is binding with $\mu > 0$,

$$f_2 G_2(\delta_2^E)\left[h_y(y_2^E,\omega_2) - 1\right](1-\lambda_E) = \mu\left[h_y(y_2,\omega_1) - h_y(y_2,\omega_2)\right] > 0$$

Using the FOC with respect to c_2, we can replace μ in the previous condition to get

$$\frac{f_2 G_2(\delta_2^E)\left[h_y(y_2^E,\omega_2) - 1\right]}{h_y(y_2,\omega_1) - h_y(y_2,\omega_2)} = \frac{\lambda_E}{1-\lambda_E} f_2 G_2(\delta_2^E) - f_2 g_2(\delta_2^E)\left[\delta_2^E - y_2^E + h(y_2^E,\omega_2)\right] \ . \quad (42)$$

Combining the FOCs with respect to c_1 and c_2 and inserting $y_1^E - h(y_1^E,\omega_1) = \delta^*(\omega_1)$ gives

$$\frac{\lambda_E}{1-\lambda_E} = \frac{f_1 g_1(\delta_1^E)\left[\delta_1^E - \delta^*(\omega_1)\right] + f_2 g_2(\delta_2^E)\left[\delta_2^E - y_2^E + h(y_2^E,\omega_2)\right]}{f_1 G_1(\delta_1^E) + f_2 G_2(\delta_2^E)} \ .$$

Combining the last equation and equation (42), we can eliminate the Lagrangian parameter λ^E and rearrange terms to obtain the optimality condition (25), according to which the optimal level of y_2^E equates the marginal deadweight losses from distortions at both margins. □

B Supplementary material

Appendix B provides additional formal results, comments and figures that complement the arguments and theoretical results in the main text. The proofs to all formal results are available upon request.

B.1 Optimal income taxes for other welfare weights

In the following, I define two further sets of social weight sequences, for which specific properties of the optimal allocation can be identified unambiguously (see Propositions 4, 5, 6 and 7 below).

Definition 2. *The sets of weight sequences $\mathcal{A}^N$ and $\mathcal{A}^D$ are defined as follows:*

(i) Set $\mathcal{A}^D$ contains all sequences $\alpha \in \mathcal{A}^\chi$ such that $\alpha_{j+1} \leq \beta_j^D(\alpha_j)$ for all $j \in J_{-n}$, with a strict inequality for at least one $j \in J_{-n}$.

(ii) Set $\mathcal{A}^N$ contains all sequences $\alpha \in \mathcal{A}^\chi$ such that $\alpha_{j+1} \in \left[\beta_j^D(\alpha_j), \beta_j^U(\alpha_j)\right]$ for all $j \in J_{-n}$.

The construction of both subsets can again be illustrated using Figure 1 in Section 5. As before, assume that the functions β_j^D and β_j^U were identical for all $j \in J_{-n}$. For social weights in $\mathcal{A}^D$, all weight-pairs are located in region I at the lowest part of Figure 1. This case represents a social planner with large concerns for local redistribution between all pairs of workers with adjacent skill types, including the workers with the lowest skill types. A limit case is given by the Rawlsian welfare function.For social weights in $\mathcal{A}^N$, all weight-pairs are located in region II in the center of Figure 1. This case represents a social planner with rather limited concerns for redistribution between all groups of workers.

Proposition 4. *For any $\alpha \in \mathcal{A}^D$, optimal output y^α is*

- *downwards distorted at the intensive margin in all skill groups $j \in J_{-n}$,*
- *downwards distorted at the extensive margin in all skill groups if α_1 is below some threshold $\gamma^D > 1$.*

By Proposition 4, optimal labor supply by all except the highest-skilled workers is downwards distorted at the intensive margin for any social weights in the set $\mathcal{A}^D$. For social weight in this set, the "central result of optimal income taxation" (Hellwig 2007) is hence valid: The optimal marginal tax is strictly positive for all income levels below the top income y_n^α. At the extensive margin, labor supply in all skill groups is downwards distorted if α_1, the weight of the lowest-skilled workers, is below some threshold γ^D. Note that the threshold γ^D is strictly higher than in the solution to the relaxed problem characterized in Lemma 5 (where it is equal to 1). If and only if $\alpha_1 < \gamma^D$, the optimal participation taxes are strictly positive at all income levels as well: the optimal income tax is a *Negative Income Tax.*

Proposition 5. *For any $\alpha \in \mathcal{A}^N$, optimal output y^α is*

- *undistorted at the intensive margin in all skill groups;*
- *upwards distorted at the extensive margin in each skill group j such that $\alpha_j > 1$.*

By Proposition 5, optimal labor supply is undistorted at the intensive margin in all skill groups for any social weights in the set $\mathcal{A}^N$. This does not imply identical tax *levels* for all agents.[75] The optimal allocation can be decentralized by a piecewise horizontal tax schedule, however, in stark contrast to the before-mentioned "central result of optimal tax theory". The optimal distortions at the extensive margin in each skill group depend on whether the social weight associated to this group is below or above the average weight of 1. If and only if $\alpha_1 > 1$, the least-productive workers benefit from a negative participation tax. Hence, the labor supply distortions in the solutions to the relaxed problem and the full problem of optimal taxation are identical (see Lemma 5).

Proposition 3 above provides conditions under which the marginal welfare weights belong to set $\mathcal{A}^U$ for some *regular* combinations of Ψ and γ, giving rise to the optimality of an *EITC* with negative marginal taxes and negative participation taxes. In the following, I clarify the conditions under which some welfare functions with standard properties give rise to welfare weights in the sets $\mathcal{A}^D$ and $\mathcal{A}^N$.

Proposition 6. *There is a number $a^D > 1$ such that, if $\frac{\omega_{j+1}}{\omega_j} < a^D$ for all $j \in J_{-n}$, there exist regular combinations of Ψ and γ for which $\bar{\alpha} \in \mathcal{A}^D$.*

Proposition 7. *There exist regular combinations of Ψ and γ for which $\bar{\alpha} \in \mathcal{A}^N$.*

By Proposition 6, there exist well-behaved welfare functions for which a *Negative Income Tax* is optimal whenever the relative distance between all pairs of adjacent skill types is sufficiently small. By Proposition 7, there exist well-behaved welfare functions for which optimal labor supply is undistorted at the intensive margin in all skill groups whenever Conditions 1, 2 and 3 are satisfied.

B.2 Construction of weight sequences for simulations

Figure 3 below graphically depicts for all agents with incomes up to \$100,000 the welfare weight sequences α^A and α^B, which are used in the numerical simulation in Section 7. In particular, it plots the welfare weight α_j associated to each skill group $j \in J$ (on the vertical axis) against the skill-specific gross income y_j^α in the optimal allocation (on the horizontal axis). As can be seen, both sequences α_A and α_B are monotonically decreasing.

The blue solid line shows sequence α^A, which is constructed by setting $\alpha_j^A = 1.05$ for all $j \in \{1, \ldots, 41\}$, i.e., all workers with incomes below $y_{41}^A = \$7,022$. Note that this

[75] In contrast, the tax schedule will always be increasing over some income range. Additionally, it may also be decreasing over some (low) income range.

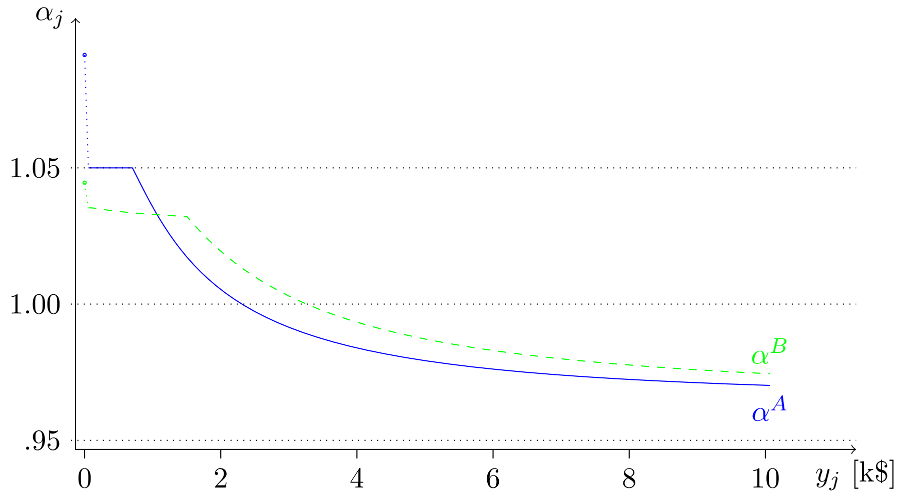


Figure 3: Welfare weight sequences α^A and α^B.

weight is above the fixed-point $\bar{\beta}_j$, implying that the upward IC constraint between skill group j and $j+1$ is violated for each $j \leq 40$ (see Lemma 6). For all $j > 41$, I set weight $\alpha_j^A = \max\left\{\beta_{j-1}^D(\alpha_{j-1}^A), \underline{\beta}_j, \ldots, \underline{\beta}_{n-1}\right\}$. By construction, α^A is hence an element of set $\mathcal{A}_{41}^U$. The weight α_0^A on the unemployed is set to have an average weight of 1 in the optimal allocation. It is slightly larger than 1.09.

The green dashed line shows sequence α^B, which is constructed just as sequence γ^k in the proof of Lemma 24. For this purpose, I first define the auxiliary function $\hat{\beta}_j^k := \max\left\{\bar{\beta}_j, \ldots, \bar{\beta}_{k-1}\right\}$. I then construct α^B by setting $\alpha_1^B = \hat{\beta}_1^{\bar{k}}$, $\alpha_j^B = \min\left\{\hat{\beta}_j^{\bar{k}}, \beta_{j-1}^U(\gamma_{j-1})\right\}$ for all $j \in \left\{2, \ldots, \bar{k}\right\}$ and $\alpha_j^B = \max\left\{\beta_{j-1}^D(\alpha_{j-1}^B), \underline{\beta}_j, \ldots, \underline{\beta}_{n-1}\right\}$ for all $j \in \left\{\bar{k}+1, \ldots, n\right\}$. Threshold $\bar{k}$ is set to 53, which is the highest number in J such that the conditions in Proposition 3 are satisfied, given ϕ_j^k equal to $(\alpha_1^B - \alpha_j^B)/(\alpha_1^B - 1)$ and δ_j^k equal to the level of δ_j in the relaxed problem's solution for sequence α^B. Note that $y_{53}^B = \$15,016$. Finally, I again set $\alpha_0^B \approx 1.045 > \alpha_1^B \approx 1.035$ to have an average weight of 1.

B.3 Elasticity-based condition for violated IC constraints

Equation (19) in Section 5 provides a condition under which the solution to the relaxed problem violates the upward IC constraint between two adjacent skill groups, expressed in terms of labor supply elasticities and welfare weights. Note first that the participation threshold δ_j in skill group j in the relaxed problem's solution is defined by equation (16). Inserting this equation into the upward IC constraint (33) between skill groups j and $j+1$, we get

$$\begin{aligned} \frac{\alpha_{j+1}-1}{A_{j+1}(\delta_{j+1}^{\alpha R})} - \frac{\alpha_j - 1}{A_j(\delta_j^{\alpha R})} &\leq -\delta^*(\omega_{j+1}) + \delta^*(\omega_j) + h(y_{j+1}^{\alpha R}, \omega_j) - h(y_{j+1}^{\alpha R}, \omega_{j+1}) \\ &= \left[y_j^{\alpha R} - h(y_j^{\alpha R}, \omega_j)\right] - \left[y_{j+1}^{\alpha R} - h(y_{j+1}^{\alpha R}, \omega_j)\right] =: \mathcal{B}_j \ , \end{aligned}$$

where I use that $\delta_k^* = y_k^{\alpha R} - h(y_k^{\alpha R}, \omega_k)$ for each $k \in J$. With $\eta_k(c^{\alpha R}, y^{\alpha R}) = A_k(\delta_k^{\alpha R})$, this gives equation (19).

To show how $\mathcal{B}_j$ can be rewritten in terms of elasticities, I define $\hat{y}(\omega) = y \in \mathbb{R} : h_y(\hat{y}(\omega), \omega) = 1$. Then, there is a unique number $\hat{\omega} \in (\omega_j, \omega_{j+1})$ such that

$$\begin{aligned} \mathcal{B}_j &= \left[h(y_{j+1}^{\alpha R}, \omega_j) - y_{j+1}^{\alpha R}\right] - \left[h(y_j^{\alpha R}, \omega_j) - y_j^{\alpha R}\right] \\ &= \int_{\omega_j}^{\omega_{j+1}} \left[h_y(\hat{y}(\omega), \omega_j) - h_y(y_j^{\alpha R}, \omega_j)\right] d\omega \\ &= \left[h_y(\hat{y}(\hat{\omega}), \omega_j) - h_y(y_j^{\alpha R}, \omega_j)\right] (y_{j+1}^{\alpha R} - y_j^{\alpha R}) \,. \end{aligned}$$

Using the definitions of $\varepsilon_{y,\omega}$ and $\varepsilon_{y,1-T'}$, an approximation of $\mathcal{B}_j$ is given by

$$\mathcal{B}_j \approx \left[\frac{\varepsilon_{y,\omega}(y_j^{\alpha R}, \omega_j)}{\varepsilon_{y,1-T'}(y_j^{\alpha R}, \omega_j)} h_y(y_j^{\alpha R}, \omega_j) \frac{\hat{\omega} - \omega_j}{\omega_j}\right] \left(\varepsilon_{y,\omega}(y_j^{\alpha R}, \omega_j) y_j^{\alpha R} \frac{\omega_{j+1} - \omega_j}{\omega_j}\right) .$$

For $\hat{\omega} \approx \frac{\omega_{j+1} - \omega_j}{2}$, we get the expression in footnote 42. The exact form of $\mathcal{B}_j$ is given by

$$\mathcal{B}_j = \int_{\omega_j}^{\hat{\omega}} \frac{\varepsilon_{y,\omega}(\hat{y}(\omega), \omega)}{\varepsilon_{y,1-T'}(\hat{y}(\omega), \omega_j)} \frac{h_y(\hat{y}(\omega), \omega_j)}{\omega} d\omega \int_{\omega_j}^{\omega_{j+1}} \varepsilon_{y,\omega}(\hat{y}(\omega), \omega) \frac{\hat{y}(\omega)}{\omega} d\omega$$

for a unique number $\hat{\omega} \in (\omega_j, \omega_{j+1})$.

A similar equation as (19) can be provided for the violation of the downward IC constraint (29). In particular, after inserting (16) and $\delta_k^* = y_k^{\alpha R} - h(y_k^{\alpha R}, \omega_k)$, I find that the downward IC constraint between skill groups j and $j+1$ is violated if

$$\frac{\alpha_{j+1} - 1}{\eta_{j+1}(c^{\alpha R}, y^{\alpha R})} < \frac{\alpha_j - 1}{\eta_j(c^{\alpha R}, y^{\alpha R})} - \mathcal{B}'_j \,,$$

where $\mathcal{B}'_j = \left[y_{j+1}^{\alpha R} - h(y_{j+1}^{\alpha R}, \omega_{j+1}\right] - \left[y_j^{\alpha R} - h(y_j^{\alpha R}, \omega_{j+1}\right] > 0$.

B.4 Elasticity-based result for optimal phase-in range

By Proposition 3, the optimal allocation may upward distortions at the intensive margin for skill groups 2 to k given well-behaved welfare functions if a set of conditions on the primitives of the model (joint type distribution, type set, effort cost function) is satisfied. As argued in Section 5, there exists a unique threshold group $\bar{k}$ such that these conditions are met if and only if $k \leq \bar{k}$. The location of threshold $\bar{k}$ depends on the type distribution and the effort cost function. Unfortunately, these quantities are not directly observable. They are related to quantities such as labor supply elasticities and skill shares that can be estimated empirically.

In the following, I derive a necessary condition for the optimality of negative marginal taxes in skill groups 2 to k that is expressed mainly in terms of these observable quantities. Thereby, I also provide an upper bound on threshold $\bar{k}$ that can be computed from empirical estimates. The derivation of this result makes use of the following condition, which imposes a lower bound on the income elasticity with respect to the retention rate and an upper bound on the income elasticity with respect to the skill type (complementing Condition 3).

Condition 4. *There are two numbers* $\mu_3 \in (0, \mu_1]$ *and* $\mu_4 \in [\mu_2, \infty)$ *such that, for each* $y > 0$ *and* $\omega > 0$, $\varepsilon_{y,1-T'}(y, \omega) \geq \mu_3$ *and* $\varepsilon_{y,\omega}(y, \omega) \leq \mu_4$.

Lemma 25. *Let Condition 4 be satisfied. For any* $k \in \{2, \ldots, n-1\}$, *there can only exist regular combinations* (Ψ, Γ) *for which* $\bar{\alpha} \in \mathcal{A}_k^U$ *if*

$$\sum_{j=1}^{k} f_j + \sum_{j=k+1}^{n} f_j \left[1 - G_j(\delta_j^k)\right] < \frac{1 - \underline{\beta}_{n-1}}{\bar{\beta}_{k-1} - 1} \sum_{j=m^k}^{n} f_j G_j(\delta_j^k) \ ,$$

where vector $(\delta_j^k)_{j=1}^n$ *is given as in Proposition 3 and threshold* m^k *satisfies*

$$\ln\left(\frac{\omega_{m^k}}{\omega_k}\right) > \frac{\mu_3}{2\mu_4^2\ \eta_k} \ \frac{\bar{\beta}_{k-1} - 1}{y_{m^k}^{\alpha R}} \ .$$

By Lemma 25, the share of workers with upward distortions at the intensive margin (skill groups k and lower) has to be small enough, compared with the share of highly skilled workers in the optimal allocation. Thereby, it supports the interpretation of Proposition 3 provided in Section 5. More precisely, Lemma 25 first compares the combined shares of unemployed agents and workers in skill group 1 to k on the one hand, and the share of workers in a threshold skill group m^k and higher, weighted by a factor that only depends on the fixed-points identified in Lemma 6, on the other hand. Second, Lemma 25 provides an upper bound on the relative distance between k and the threshold skill group m^k that depends on the intensive-margin and extensive-margin labor elasticities and one of the fixed-points mentioned above. (It should be noted that the fixed-points $\bar{\beta}_{k-1}$ and $\underline{\beta}_{n-1}$ are functions of the labor elasticities and the relative distances between adjacent skill types themselves.) It is easy to see that these conditions can only be satisfied if skill level ω_k is smaller enough, both in terms the share of workers with skill ω_k and lower and in terms of the ratio between skill ω_k and the average skill in the population. Put differently, there exists a unique threshold $\tilde{k}$ such that the conditions in Lemma 25 are satisfied if and only if $k < \tilde{k}$. This threshold $\tilde{k}$ is hence an upper bound on the critical value $\bar{k}$ that is implied by Proposition 3.

Finally, note that the threshold group m^k (for each k) and the critical values $\bar{k}$ and $\tilde{k}$ can be computed explicitly if one has more exact information about the labor supply elasticities, the skill distribution and the underlying primitives of the model. In particular, this is done numerically for the calibrated model in Section 7, and analytically for the example considered in Subsection B.5 below (for the limit case where the discrete skill set converges to an interval).

B.5 Limit result for continuous skill sets

All theoretical results have been derived under the assumption that skill set Ω is given by some discrete set $\{\omega_1, \ldots, \omega_n\}$, while the fixed cost set Δ is given by an interval $\left[\underline{\delta}, \bar{\delta}\right]$. In particular, the formal proofs for Lemma 6 and Propositions 2 and 3 exploit the discreteness of Ω. Besides, the results are relatively general as I have not imposed any functional forms assumptions (except for the quasi-linearity of utility in consumption). Still, one might ask whether the central insight of this paper – the potential optimality of an *EITC* with negative marginal taxes – extends to a model with a continuous skill set as in Jacquet et al. (2013) and many other related papers. Under the level of generality maintained in the analysis above, this question cannot be clarified.

In the following, however, I show the optimality of negative marginal taxes indeed extends to a continuous skill set for an example with simple, commonly used functional forms, however. In particular, I impose the following set of assumptions.

Condition 5. *The economy has the following properties:*

(i) The effort cost function is given by $h(y, \omega) = \frac{1}{1+1/\sigma}\left(\frac{y}{\omega}\right)^{1+1/\sigma}$ *with* $\sigma > 0$*,*

(ii) the skill space is given by the finite set $\{\omega_1, \omega_2, \ldots, \omega_n\}$ *with constant relative distances* $\frac{\omega_{j+1}}{\omega_j} = a > 1$ *for each* $j \in J/\{n\}$*,*

(iii) the fixed cost space is given by the interval $\left[0, \bar{\delta}\right]$*, and*

(iv) for each $j \in J$*, the conditional distribution* G_j *of fixed costs is given by a uniform distribution on* $\left[0, \bar{\delta}_j\right]$*, with* $\bar{\delta}_j \leq \bar{\delta}$ *for all* $j \in J$*.*

The first two parts of Condition 5 have already been imposed for the calibration in Section 7. Part (i) implies that the elasticity of output (or gross income) y with respect to the retention rate is equal to parameter σ for all workers and all admissible tax functions. Part (ii) plays a crucial role for the following exercise, because parameter a can be seen as a measure of how "dense" the skill set is. In particular, it allows to consider the limit case where a converges to 1, i.e., the skill set converges to an interval. Parts (iii) and (iv) imply that fixed cost types are uniformly distributed in each skill group, in contrast to the logistic distributions used in Section 7. Importantly, this assumption allows me to obtain closed-form expressions of the optimal skill-specific participation thresholds $\delta_1^{\alpha R}, \ldots, \delta_n^{\alpha R}$ and the fixed-points $\underline{\beta}_j$, $\bar{\beta}_j$ established in Lemma 6. Ultimately, this simplification enables me to derive analytical limit results. The drawback of part (iv) is that participation elasticities are larger than 1 and identical across all skill groups (under the approximated US income tax), which is at odds with the available empirical evidence. Besides, Condition 5 allows to math all other empirical moments targeted in Section 7. Note also that Conditions 1, 2 and 3 in Section 4 are satisfied for all parameter constellations under Condition 5.

In the following, I am interested in studying whether an *EITC* with negative marginal taxes remains optimal for some well-behaved welfare function if the relative distance a

between adjacent skill types converges to 1, i.e., the discrete skill set $\Omega = \{\omega_1, \ldots, \omega_n\}$ converges to the interval $[\omega_1, \omega_n]$. Formally, I want to investigate whether there exist regular combinations (Ψ, γ) such that $\bar{\alpha} \in \mathcal{A}_k^U$ for some $k \in J$ in the limit case $a \to 1$. Additionally, I am interested in whether the skill range (and the corresponding income range) with potentially optimal marginal taxes shrinks, or even disappears, in this limit case. For this purpose, I reconsider Lemma 25, which was established in Appendix B.4, for an economy that satisfies Condition 5.

Lemma 26. *Let Assumption 5 be satisfied and consider the limit case $a \to 1$. For any $\sigma > 0$ and $\omega_k \in (\omega_1, \omega_n)$, there can only be regular combinations of (Ψ, γ) such that $\bar{\alpha} \in \mathcal{A}_k^U$ if*

$$\sum_{j=1}^{k} f_j + \sum_{j=k+1}^{n} f_j \left[1 - G_j(\delta_j^k)\right] < \sum_{j=m^k}^{n} f_j G_j(\delta_j^k) \,, \tag{43}$$

where m_k is defined implicitly by $y_{m^k}^{\alpha R} = 2 y_k^{\alpha R}$.

By Lemma 26, the optimal allocation can involve upwards distortions at the intensive margin for a substantial set of low-skill workers even in the limit case where Ω converges to a continuous set. To see this, consider some skill type $\omega_k \in (\omega_1, \omega_n)$. Lemma 25 implies that the optimal allocation can involve upwards distortions for all workers with skills in $(\omega_1, \omega_k]$ if and only if the share of high-skill workers with incomes above $2y_k^{\alpha R}$ is larger than the combined share of unemployed agents and low-skill workers with incomes below $y_k^{\alpha R}$. Hence, the optimal income tax can only involve negative marginal taxes for, first, a minority of agents and, second, for agents with earned incomes below the population average. There is no doubt that these conditions limit the optimal phase-in range, i.e., subset of agents facing optimally negative marginal taxes, making more transparent the restrictions that are already present in Proposition 3. However, the potential optimality of negative marginal taxes clearly does not vanish or shrink to a economically irrelevant subset of low-skill workers. Interestingly, this insight does not depend on parameter σ, which determines the level of the (intensive-margin) elasticity of income with respect to the retention rate.

B.6 Illustration of labor supply distortions

In Subsection 3.2, I formally define labor supply distortions at the intensive margin and at the extensive margin. The following Figures 4 and 5 illustrate these definitions graphically. In each figure, point A marks the initial bundle (c^i, y^i) allocated to agent i. The sets of hypothetical deviations are given by the solid lines through A and B. The indifference curves of Agent i are given by the union of the dashed line and point Z (in figure 4) and the union of the dashed line and point A (in figure 5), respectively, corresponding to the discontinuity in i's utility due to the fixed cost δ^i.

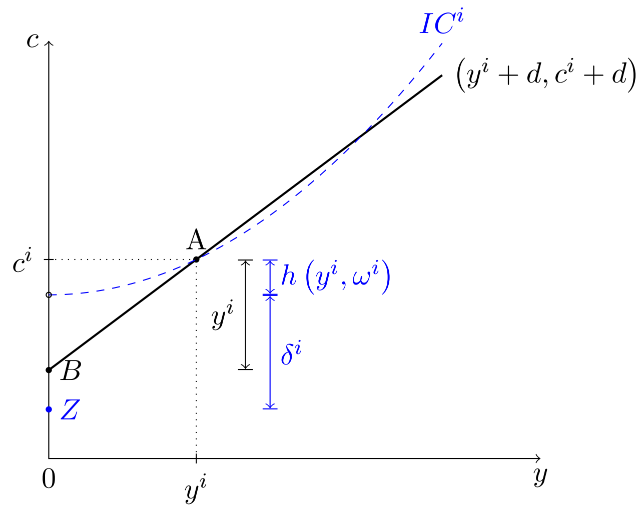


Figure 4: Labor supply distortions, example 1.

In Figure 4, i's initial output is strictly positive, $y^i > 0$. In point A, the slope of the indifference curve is below 1, the marginal rate of substitution. Hence, i's utility could be increased by moving slightly upwards the solid line. Alternatively, i's utility could also be increased by jumping downwards to point B, where output provision is zero. Hence, i's labor supply is both downwards distorted at the intensive margin and upwards distorted at the extensive margin.[76]

In Figure 5, i does not provide output initially, $y^i = 0$. Jumping upwards to point B with positive output $y^*(\omega^i) = \arg\max_{y>0} \{y - h(y, \omega^i)\}$ would increase i's utility, as B is located above the indifference curve. Hence, i's labor supply is downwards distorted at the extensive margin.

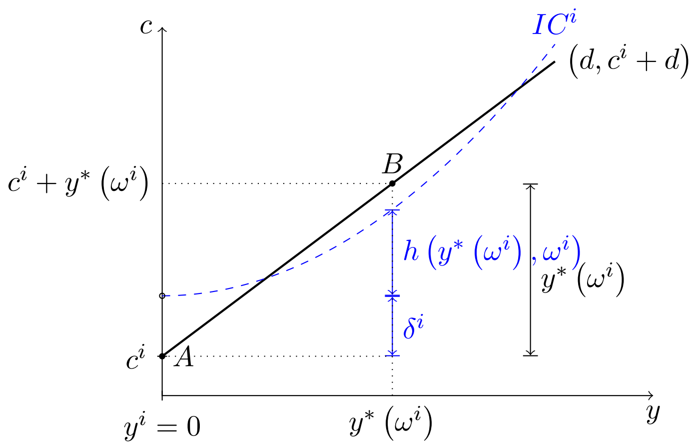


Figure 5: Labor supply distortions, example 2.

[76] In a model without fixed costs as in, e.g., Mirrlees (1971), this would be impossible by construction.

B.7 Construction, decomposition and illustration of deadweight loss

Following the literature, I formally define the deadweight loss in an implementable allocation (c, y) as

$$\begin{aligned} DWL(c,y) \;&=\; \sum_{j=1}^{n} f_j \int_{\underline{\delta}}^{\delta^*(\omega_j)} g_j(\delta)\,[\delta^*(\omega_j)-\delta]\,d\delta \\ &\quad -\sum_{j=1}^{n} f_j \int_{\underline{\delta}}^{\delta_j} g_j(\delta)\,[y_j - h(y_j,\omega_j)-\delta]\,d\delta \,. \end{aligned}$$

By construction, the deadweight loss measures the maximum increase in the difference between the consumption possibilities that result from providing labor and the total cost of providing labor that can be achieved by moving from allocation (c, y) to some other feasible allocation. As usual, $DWL(c, y)$ is minimized and equal to zero if labor supply in all skill groups is undistorted at both margins, i.e., if $h_y(y_j, \omega_j) = 1$ and $\delta_j = \delta^*(\omega_j) = \max_{y>0} y - h(y, \omega_j)$ for every $j \in J$.

The overall deadweight loss can also be decomposed as

$$\begin{aligned} DWL(c,y) \;&=\; \sum_{j=1}^{n} f_j G_j(\delta_j)\,[\delta^*(\omega_j)-y_j+h\,(y_j,\omega_j)] \\ &\quad +\sum_{j=1}^{n} f_j \int_{\delta_j}^{\delta^*(\omega_j)} g_j(\delta)\,[\delta^*(\omega_j)-\delta]\,d\delta \,, \end{aligned}$$

where the first term captures the deadweight loss from distortions at the intensive margin and the second term captures the deadweight loss from distortions at the extensive margin (across all skill groups).

Figure 6 below illustrates the deadweight loss from distortions in skill group j and its decomposition graphically. It depicts the quantity $G_j(\delta_j)$ of labor of skill type ω_j, and the labor supply S_j and labor demand D_j (measured in mass of workers) for an allocation that involves bundle (y_j, δ_j). D_j^* depicts the labor demand that would result without distortions at the intensive margin, i.e., if each worker would provide efficient output. In particular, the figure depicts a case where labor supply in skill group j is downwards distorted at both margins, $h_y(y_j, \omega_j) < 1$ and $\delta_j < y_j - h(y_j, \omega_j) < \delta^*(\omega_j)$. The red shaded area ($L_i$) depicts the efficiency loss due to intensive-margin distortions, the blue-shaded area (L_e) depicts the efficiency loss due to extensive-margin distortions in this skill group.

B.8 Monotonicity of social weights

In Subsection 3.4, the endogenous marginal social weights are defined in equations (9) and (10). In the following, I assume that all type-specifid weight are equal, i.e., that

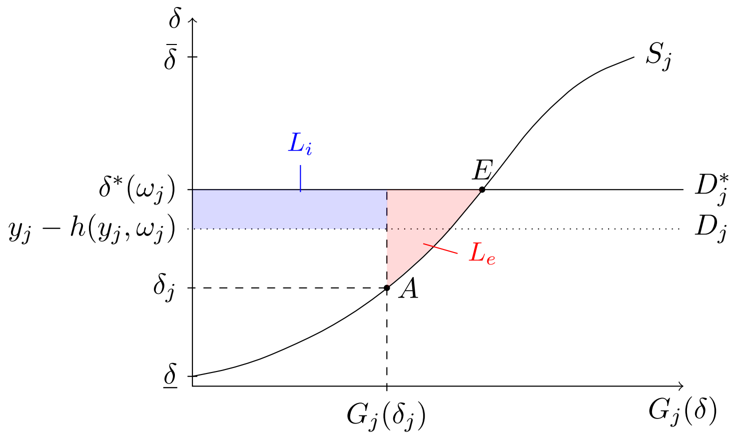


Figure 6: Illustration of deadweight loss in skill group j.

$\gamma(\omega, \delta) = 1$ for all $(\omega, \delta) \in \Omega \times \Delta$. Then, concavity of Ψ ensures that $\bar{\alpha}_0 > \bar{\alpha}_j$ for all $j \in J$. For $j \geq 1$, however, the endogenous weight sequence $\bar{\alpha}$ is only ensured to be decreasing if additional conditions on the joint type distribution K are met.

Lemma 27. *If Ψ is strictly concave and γ is constant over $\Omega \times \Delta$, G_j dominates G_{j+1} in the sense of first-order stochastic dominance for all $j \in J_{-n}$ and Condition 1 holds, $\bar{\alpha}_j > \bar{\alpha}_{j+1}$ for all $j \in J_{-n}$ in all implementable allocations.*

In the following, I provide a simple example to demonstrate that the concavity of Ψ per se does not guarantee decreasing social weights.

Example 1. *Assume that $n > 2$, $\omega_1 = 1$, $\omega_2 = 3/2$, $\underline{\delta} = 0$, $\bar{\delta} = 10$, $h(y, \omega) = \frac{1}{2}\left(\frac{y}{\omega}\right)^2$, $\Psi(x) = x^{1/2}$, $\gamma(\omega, \delta) = 1$ for all $(\omega, \delta) \in \Omega \times \Delta$, $g_1(\delta) = 0.1$ for all $\delta \in \Delta$, $g_2(\delta) = \varepsilon$ for $\delta \in [0, 1]$ and $g_2(\delta) = \frac{1-\varepsilon}{9}$ for $\delta \in (1, 10]$.*

Note that function Ψ is strictly concave. Fixed costs types are uniformly distributed in skill group 1, and piecewise uniformly distributed in skill group 2. For ε below (above) 0.1, G_2 dominates (is dominated by) G_1 in the sense of first-order stochastic dominance.

Consider the allocation (c', y') with $c'_0 = 0.1$, $(c'_1, y'_1) = (1.1, 1)$, $(c'_2, y'_2) = (9/4 + .1, 9/4)$, $\delta'_1 = 1/2$ and $\delta'_2 = 9/4$. Note that this allocation satisfies both IC constraints between workers with skill types ω_1 and ω_2 with strict inequalities. The social weights in this allocation are given by $\bar{\alpha}_0(c'_0) \approx 1.581/z$, $\bar{\alpha}_1(\delta'_1, c'_0) \approx .917/z$ and $\bar{\alpha}_2 \approx (1.265 + 44.272\varepsilon)/((1 + 71\varepsilon)z)$, where $z > 0$ is again a normalizing parameter. I find that $\bar{\alpha}_2 > \bar{\alpha}_1$ if and only if ε is below some threshold $\hat{\varepsilon} \approx 0.0167$. In this example, the social weights are hence locally increasing if G_2 first-order stochastically dominates G_1 "sufficiently much". Loosely speaking, the workers in skill group 2 are on average worse off than the workers in skill group 1 in this case, because they have on average much higher fixed costs.

B.9 Validity of Conditions 1 and 2 for specific functions

The theoretical results are valid whenever the skill-specific fixed cost distributions satisfy Conditions 1 and 2. In the following, I show that these conditions indeed hold for many commonly used functional forms. First, Condition 1 (i) requires the fixed cost distribution G_j to be log-concave for each $j \in J$.

Observation 1. *For any $j \in J$, Condition 1* (i) *is satisfied if G_j is given by*

(a) a uniform distribution on $\left[\underline{\delta}_j, \bar{\delta}_j\right]$;

(b) a logistic distribution of the functional form (28) *with location parameter $\psi_j \in \mathbb{R}$ and scale parameter $\rho_j \notin 0$;*

(c) a Pareto distribution with scale parameter (minimum value) $\underline{\delta}_j > 0$ and shape parameter $k_j > 0$;

(d) a log-normal distribution with location parameter $\xi_j \in \mathbb{R}$ and scale parameter $\sigma_j > 0$;

(e) a normal distribution with mean $\xi_j \in \mathbb{R}$ and standard deviation $\sigma_j > 0$.

Condition 1 (ii) refers to the co-variation of distributions G_j and G_{j+1} for each pair of skill groups j and $j+1$. In particular, it assumes that the *cdf* hazard rates can be monotonically ordered for each $\delta \in \Delta$, $A_j(\delta) \geq A_{j+1}(\delta)$. In general, this assumption is neither stronger nor weaker than the assumption that G_{j+1} first-order stochastically dominates G_j. Within each of the families of distribution functions considered here, however, both properties are equivalent.

Observation 2. *For any $j \in J$, Condition 1* (ii) *is satisfied and $G_{j+1}(\delta) \geq G_j(\delta)$ for all $\delta \in \Delta$ if the fixed cost distribution G_j and G_{j+1} are given by*

(a) uniform distributions with upper endpoints $\bar{\delta}_j \geq \bar{\delta}_{j+1}$ and lower endpoints $\underline{\delta}_j = \underline{\delta}_{j+1}$;

(b) logistic distributions of form (28) *with location parameters $\psi_j \geq \psi_{j+1}$ and scale parameter $\rho_j = \rho_{j+1}$, or scale parameters $\rho_j \geq \rho_{j+1}$ and location parameters $\psi_j = \psi_{j+1}$;*

(c) Pareto distributions with shape parameters $0 < k_j \leq k_{j+1}$ and scale parameters (minimum values) $\underline{\delta}_j = \underline{\delta}_{j+1} > 0$;

(d) log-normal distributions with location parameters $\xi_j \geq \xi_{j+1} \in \mathbb{R}$ and scale parameters $\sigma_j = \sigma_{j+1} > 0$;

(e) normal distributions with expected values $\xi_j \geq \xi_{j+1} \in \mathbb{R}$ and standard deviations $\sigma_j = \sigma_{j+1} > 0$.

Finally, Condition 2 requires the *pdf* hazard rate $a_j(\delta_j)$ to be weakly decreasing in δ and weakly increasing in ω, but only at a sufficiently small rate compared to the derivative of the *cdf* hazard rate $A_j(\delta_j)$.

Observation 3. *For any $j \in J$, Condition 2 is satisfied*

(a) for all $\delta \in \Delta$ if G_j and G_{j+1} are given by uniform distributions with upper endpoints $\bar{\delta}_j \geq \bar{\delta}_{j+1}$ and identical lower endpoints $\underline{\delta}_j = \underline{\delta}_{j+1}$;

(b) for all $\delta \in \Delta$ if G_j and G_{j+1} are given by logistic distributions of form (28) *with location parameters $\psi_j \geq \psi_{j+1}$ and identical scale parameters $\rho_j = \rho_{j+1}$, or with scale parameters $\rho_j \geq \rho_{j+1}$ and identical location parameters $\psi_j = \psi_{j+1}$;*

(c) for all δ below some threshold level $z_j > \xi_j$ if G_j and G_{j+1} are given by normal distributions with means $\xi_j \geq \xi_{j+1}$ and identical standard deviations $\sigma_j = \sigma_{j+1} > 0$.

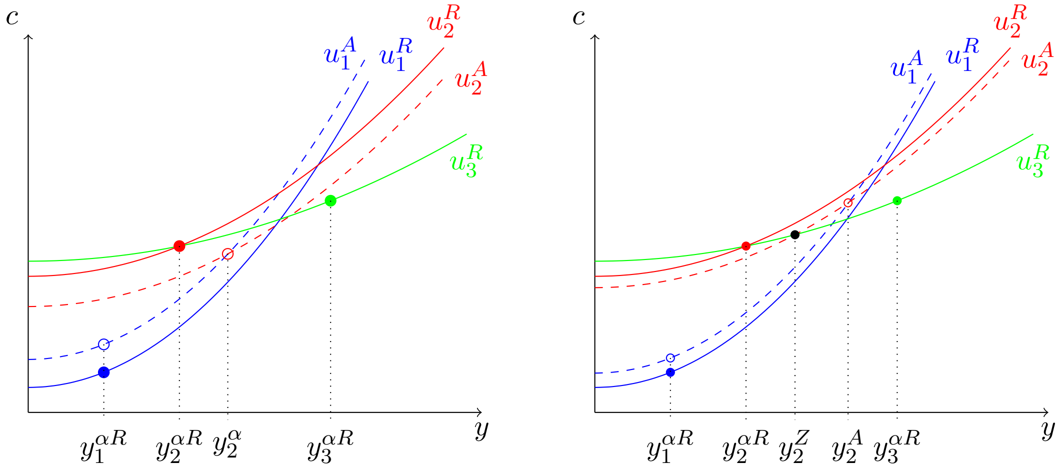
Condition 2 is not satisfied if G_j and G_{j+1} are given by Pareto or log-normal distributions. In particular, Condition 2 (i) is violated: the *pdfs* of these distribution functions are not log-concave for any combination of parameters. It is possible, however, to provide a relaxed version of the condition that (a) is satisfied for these distribution functions in many cases and (b) continues to ensure the validity of Lemma 6 and all subsequent results. The details are available upon request.

B.10 Illustration: Optimally binding upward IC constraints

By Proposition 2, optimal labor supply in all skill groups $j \in \{2, \ldots, k\}$ is upwards distorted at the intensive margin for every social weight in the set $\mathcal{A}_k^U$. In particular, the formal proof in Appendix A shows that the upward IC constraint between skill groups j and $j+1 \leq k$ will always be binding in the optimal allocation. With respect to the IC constraints between skill groups k and $k+1$, there are multiple possible constellations. In particular, the downward IC constraint between workers in both skill groups may be binding or slack in the optimal allocation. In both cases, however, labor supply in skill group k is upwards distorted at the intensive margin.

Figure 7 illustrates both cases in panels 7(a) and 7(b) for the case $k = 2$. In both panels, the filled circles mark the bundles allocated to workers with skill types ω_1, ω_2 and ω_3 in the solution to the relaxed problem. The indifference curves corresponding to these bundles are drawn as solid lines. As can be seen, workers with skill type ω_1 prefer the bundle $(c_2^{\alpha R}, y_2^{\alpha R})$ to the bundle $(c_1^{\alpha R}, y_1^{\alpha R})$, while workers with skill type ω_3 are indifferent between their own bundle and the bundle designed for workers with skill type ω_2.

Consider an intermediate problem A that takes into account only the IC constraints between the workers with the two lowest skill types (see proof to Proposition 2). The solution to this problem is represented by the empty circles and the corresponding dashed indifference curves for workers with skill types ω_1 and ω_2. As can be seen, the utility of workers in skill group 1 is higher than in the solution to the relaxed problem, while the utility of workers in skill group 2 is lower. The output provided by workers in skill group 2 is strictly upwards distorted, $y_2^A > y_2^{\alpha R}$. In the case depicted in the left panel,



(a) The case with a slack downward IC constraint

(b) The case with a binding downward IC constraint

Figure 7: Illustration of binding upward IC constraint for $\alpha \in \mathcal{A}^U$

the solution to the intermediate problem A satisfies the downward IC constraint between workers in skill groups 2 and 3. In this case, the solution to intermediate problem A also solves the non-relaxed problem of optimal taxation. In the case depicted in the right panel, the solution to intermediate problem A violates the downward IC constraint between the workers in groups 2 and 3. In the solution to the non-relaxed problem (not shown), this downward IC constraint will hence be binding. The optimal output level y_2^{α} will nevertheless be upwards distorted. More precisely, it will always be located between the output levels y_2^A and y_2^Z (corresponding to the intersection point Z between the red dashed indifference curve of the workers in skill group 2 and the green indifference curve of the workers in skill group 3). Which of the two cases prevails, depends in a non-trivial way on the joint type distribution K, the effort cost function h and the complete sequence of social weights α.