Automation and Unemployment

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Abstract. This paper studies a model of technological change in which the degree to which change is labor-saving or capital-saving is determined endogenously. It analyses the effect of minimum wages on the evolution of technology, unemployment, and the stock of capital.

1. Introduction

The present paper analyses the effect of minimum wages on the evolution of employment, technology, and capital accumulation in a framework in which the type of technological change depends on the evolution of factor-prices. It makes precise and tries to answer questions like the followings:

Does the introduction of minimum wages cause a bias towards more labor-saving and less capital-saving innovations? Can they induce a *persistent* bias in the rate of automation? What is the effect of minimum wages on the rate of capital accumulation? If minimum wages induce a bias in the rate of automation and of accumulation, what is the effect of these induced changes on the further development of employment? Do minimum wages induce a bias in the rate of automation, that kills even more jobs? Does the induced innovation bias speed up the productivity growth of labor, such that market-clearing wages catch up with minimum wages. If the system was in a steady state with full employment before the introduction of minimum wages, does it attain a new steady state below full employment? Does employment after an initial decrease, tend back to full-employment? Or does initial unemployment continue to fall? Under which conditions does the minimum wage remain binding in the long-run, under which conditions does the system move back to a laissez-faire development, i.e. does technical progress raise market clearing-wages sufficiently to make obsolete old minimum wages? Which minimum wage regime is sustainable in a society in which the sustainability of a minimal-wage depends on the size of the gap between actual wages and market-clearing wages? What is the effect of a minimum wage on labor's income and labor's share in total income?

In order to formally address these or similar questions we need a framework that allows to analyze and compare (1) a laissez-faire regime with market-clearing flexible

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wages and (2) a minimum wage regime with wages that are determined exogenously and that may or may not change over time.

If the laissez-faire regime with flexible wages prevails the framework should allow to explain the joint evolution of *factor-prices,* capital stock, and technology. If the minimum–wage regime prevails the framework should allow to explain the joint evolution of *(un)employment*, capital stock, and technology in the case of binding minimum wages.

There is a certain tradition in economic literature studying the laissez-faire regime. Ricardo, in his *Principles of Political Economy* [10], argues that the invention of new machines typically benefits capitalists more than workers and may even hurt workers by decreasing wages. He believed that the introduction of improved technics typically saves labor and makes production more capital intensive which in turn should lower wages and raise capital rentals. Later Hicks [7] argues that lower wages induce a specific type of technical change, namely one which is rather labor-using (capital-saving). Putting together the two intuitions one may expect a balanced evolution of capital-saving and labor-saving innovations, as well as of wages and capital rentals.

The issue was first formally studied in the literature on induced innovation of the sixties ([5], [8], [11], [12]). In this literature the evolution of technology depends on the evolution of factor-prices and vice versa. The type of technical progress at each moment of time is determined by the choice of innovators. At any moment of time entrepreneurs can improve their technologies by increasing the productivity of labor and/or of capital (pure factor augmentation). The higher the chosen rate of labor augmentation, the smaller the possible capital augmentation. The essential hypothesis of this literature is the 'Hypothesis of Induced Innovation', which assumes that entrepreneurs choose the instantaneous rates of factor augmentation for labor and capital so as to maximize the current rate of output growth at fixed current factor employments subject to an innovation possibility frontier. Based on this Hypothesis Samuelson [11] and Drandakis and Phelps [5] describe the joint equilibrium evolution of factor-prices, factor-shares, and technology. It turns out that the evolution of factor income-shares and of the type of technology is balanced only for the case that the factors are not to good substitutes. In a companion paper [6] we have given a microeconomic foundation to the Hypothesis. In the present paper we use the model of induced innovation as the laissez-faire basis of comparison.

In this framework the effect of inventions and innovations on the well being of workers has remained a much debated issue. However, at least in continental Europe, the modern preoccupation is as much about employment (or unemployment) as about wages. Ricardo's world was one of laissez-faire with flexible wages. The shift of concern from wages to (involuntary) unemployment is natural in economies in which wages are not fully flexible. While the literature on induced change can help to correct intuitions like those expressed by Ricardo and Hicks, it cannot directly contribute to the discussion about technical progress and unemployment or answer the questions raised above, as it exclusively deals with a laissez-faire economy with completely flexible wages. However, the hypothesis of induced innovation (or its microeconomic foundation) does not depend on the way wages are determined. To address the theme of automation and unemployment we can therefore remain within the framework of this literature, only deviating from the assumption of fully flexible, market-clearing wages. Based on the Hypothesis of induced innovation we will study the joint evolution of factor-incomes, employment, capital-stock,and technology in a framework with wages that are not fully flexible.

The introduction of minimal wages has a direct effect on employment and it has an indirect effect on employment via its influence on capital accumulation and automation. The two indirect effects can be disentangled: The effect on capital accumulation can be studied in standard models of accumulation without technological progress. And the effect on automation can be studied in a model with only non-accumulating factors. In Section 2 we first study the effect of the introduction of minimum wages on the accumulation of capital and the evolution of employment in the standard model of accumulation without technological progress (or with exogenous technological progress). We will see that already minimum wages slightly higher than laissez-faire wages have extreme consequences about capital accumulation and employment. In Section 3 we consider the opposite polar case, in which capital cannot accumulate, but in which the direction of change is endogenous (in which case one may prefer to think of labor an land or of unskilled labor and skilled labor). To this end we first introduce the model of induced innovation at flexible prices. We then study the effect of a minimum wage on the direction of technological change and the evolution of unemployment in this model without capital accumulation. While the effects on accumulation and on the direction of technological change *can* be disentangled it seems nevertheless natural to consider automation in a world in which capital accumulates. We therefore finally move to the complete model with accumulation and endogenous technological change (Section 4). Section 5 summarizes.

We do not attempt in this paper to explain the occurrence or the nature of the minimum wage regime. Minimal wages may be imposed by law (as in France) or be the outcome of negotiations between unions and employers (as in Germany). What we call 'minimum wage' can also be interpreted as downward sticky net wages cum rising social charges that are part of gross wages (as prevailing in most European countries). Here we only study the consequences of minimum wages on the dynamics of automation, employment, and accumulation. We consider, however, a condition on the enforceability of a wage regime, which should be satisfied (at equilibrium) in any realistic model of wage formation.

The paper does not deal with the structural/frictional unemployment which may be caused in industries experiencing labor-saving progress (as do for instance [1] and

[4]). It is only concerned with the residual unemployment in a world were markets are sufficiently efficient to ensure that there cannot be simultaneous rationing of demand *and* supply.

2. Unemployment in the standard model of capital accumulation with exogenous technological change

Before we analyze a model in which the evolution of technology is determined endogenously, we want the understand the impact of minimum wages on the evolution of employment and the stock of capital in a standard model of accumulation with no or with exogenous progress.

We consider a standard model in continuous time. At time t labor L_t and capital K_t are used to produce an aggregate final consumption commodity $Y_t = F_t(L_t, K_t)$. The aggregate production function $F_t(\cdot)$ is linear homogeneous. Capital is produced one-to-one with final output.

2.1. No technological change. We first assume that the production function does not dependent on time. For ease of exposition we assume the simple case of a constant exogenous propensity to save (In the appendix we consider alternative saving rules). The instantaneous rate of accumulation is a constant fraction s of current aggregate output $Y_t = F(L_t, K_t)$ and there is depreciation at the rate $\delta > 0$, i.e. $\dot{K} = sF(L_t, K_t) - \delta K_t$. The rate of capital accumulation then is

$$\widehat{K}_t = \frac{sf(k_t)}{k_t} - \delta, \tag{1}$$

where $f(k) = F(1, k)$ and $k_t = \frac{K_t}{L_t}$. In the usual laissez-faire model with flexible wages, L_t is fixed at the exogenous level of labor supply $\overline{L}$. Wages and capital rentals (in terms of output) at t are the marginal productivities of the factors, $(w_t, r_t) = (F_{tL}, F_{tK} - \delta)$, where F_{tL} and F_{tK} are the first derivatives of the production function at $(\overline{L}, K_t)$. The dynamics defined through equation (1) has a globally stable steady state $k^* = \frac{K^*}{\overline{L}}$ defined by $sf(k^*) = \delta k^*$ with factor prices $(w^*, r^*) = (f(k^*) - k^* f'(k^*), f'(k^*) - \delta)$. Without technological progress capital and output approach, in the long-run, the fixed values $(K^*, F(\overline{L}, K^*))$.

We now introduce minimum wages into this standard framework. For the moment we consider the bench-mark case that starts with an initial capital stock close to the steady state of the laissez-faire economy. Later, when we analyze the general model with endogenous innovation, we will consider the dynamics starting from any initial state.

Suppose, that at $t = 0$ the system has reached a steady state in the laissez-faire with wage w^*. At $t = 0$ a minimum wage, $w_0 > w^*$, is introduced and hold constant thereafter. Firms continue to take prices as given, they employ factors such as to equate marginal products and factor-prices. Flexible capital rentals continue to guarantee that the demand for capital equals supply at each moment of time. Interest rates adapt to ensure full employment of the stock of capital. Labor demand adapts so as to make sure that the marginal productivity of labor matches the fixed minimum wage. Formally, at $t = 0$, given $\widetilde{w}$ and K_0, $\widetilde{L}_0$ is determined by $w_0 = F_L(\widetilde{L}_0, K^*) = f(\widetilde{k}_0) - f'(\widetilde{k}_0)\widetilde{k}_0$. Since $w_0 > w^*$, and since F_L is decreasing in L the minimum wage causes unemployment, i.e. $\widetilde{L}_0 < L_0 = \overline{L}$. How do employment and the stock of capital develop in the changed environment? Does the initial unemployment persist? Does it increase? Does it decline? When this question was posed to economists familiar with the neoclassical model the typical spontaneous answer was that employment and capital would presumably tend to a new steady state with constant unemployment, with the steady state level of unemployment being the smaller, the smaller the increase of wages. This intuition is natural in that it only seems to require that the system be robust against small perturbation of initial wages. Nevertheless, the first glance intuition is mistaken. Even if the minimum wage exceeds initial competitive wages only by very little, the market distortion has a large impact.

At a second glance, the reason for this 'non-robustness' with respect to the introduction of a minimum wage is very intuitive. The immediate consequence of the minimum wage is a fall of employment from $\overline{L}$ to $\widetilde{L}_0 < \overline{L}$ and therefore, given the capital stock $K_0 = K^*$, a fall of output $\widetilde{Y}_0 = F(\widetilde{L}_0, K_0)$. Consequently, the rate of accumulation in the new regime $\widehat{\widetilde{K}}_0 = s\frac{\widetilde{Y}_0}{K_0} - \delta$, is lower than it was under laissez-faire. For the present case (no technological progress, no accumulation at laissez-faire steady state) this means that the rate of accumulation is negative. The stock of capital declines. The minimum wage remains at the initial level w_0, therefore the marginal product of labor remains at the level $F_L(\widetilde{L}_0, K_0) = f(\widetilde{k}_0) - f'(\widetilde{k}_0)\widetilde{k}_0 = w_0$, too. Therefore $\widetilde{k}$ remains constant, which means that L falls at the same rate as K. Therefore, the rate of accumulation continues to fall. And it falls at a constant rate, since $\widehat{\widetilde{K}}_0 = \frac{sf(\widetilde{k}_0)}{\widetilde{k}_0} - \delta$ is constant as long as $\widetilde{k}$ remains constant (i.e. as long as the minimum wage prevails). Therefore, K and L fall at the constant rate $\frac{sf(\widetilde{k}_0)}{\widetilde{k}_0} - \delta$. The *rate* at which unemployment grows depends on the amount by which minimum wages exceed initial competitive wages.

Summarizing we have

Lemma 1. *$\widetilde{k}_t = \widetilde{k}_0$ is constant as long as the minimum wage is binding.*

Proposition 2. *Starting at the no growth steady state of a laissez-faire economy without technological change, the introduction of a minimum wage reduces initial* *employment and thereafter decreases employment and capital stock at a constant rate.*

This conclusion is rather extreme. Fixing wages only slightly above the competitive level at the laissez-faire steady state leads to a continuous fall in employment and accumulation. One may expect that endogenizing the saving rate as the result of intertemporal optimization of forward looking consumers may prevent the continuous fall in income. We show in an appendix that this is not the case.

Note that while initially the minimum wage may only slightly exceed a market clearing level, this is no longer true after a wile. The stock of capital falls continuously. Therefore market-clearing wages (full employment) decline. Thus, the gap between minimum wages and competitive wages widens continuously.

The extreme conclusion depends on the persistence of the minimum wage assumed in the bench-mark case, even if almost every worker is unemployed. If, due to the decreasing bargaining power of unions in times of high unemployment for instance, minimum wages can only be sustained up to a certain level of unemployment or a certain distance between market clearing wages and actual wages, then the economy tends back to full employment from then on. We have not endogenized minimum wages. We may however assume quite generally that a minimum wage much exceeding market clearing wages is not enforceable.

Let $w^*(K_t)$ be the market clearing wage (i.e. $L_t = \overline{L}$) at a given capital stock K_t. Then a minimal condition for the sustainability of a minimal wage is that $\frac{w^*(K_t)}{w_t}$, the ratio of the market-clearing wage at actual capital stock to the actual wage, is bounded away from zero.

Corollary 3. *Assume that there exists a $\Delta > 0$ such that in all periods $\frac{w^*(K)}{w} > \Delta$,where w is the actual wage and K the actual capital stock. Then, a fixed minimum wage is not sustainable in the long-run.*

Proof: Corollary of proposition 2, since $\frac{w^*(K_t)}{w_0}$ tends to zero, for any minimum wage w_0 exceeding the steady state market clearing wage. ■

In view of this corollary one may wonder what happens if the (unsustainable) minimum wage is gradually reduced to approach its initial steady state level again.

Proposition 4. *If the minimum wage, introduced at the laissez-faire steady state, is reduced after a while to gradually approach initial wages (laissez-faire steady state wages), then the system approaches a new steady state, with a constant level of unemployment.*

Proof: As long as the minimum wage is higher than the laissez-faire steady state level, $\widetilde{k}_t > k^*$ and the stock of capital continues to fall. Once the initial wage is reached, $\widetilde{k}_t$ is constant again at the original level k^*. Thus the capital stock is constant too. Therefore the level of employment is constant. The new stock is smaller than the initial one K^*. Since $\widetilde{k}_t$ has reached the original $k^* = \frac{K^*}{\overline{L}}$, the new level of employment is smaller than $\overline{L}$. ■

2.2. Exogenous technological change. We now add exogenous technological progress to the model of mere accumulation. Consider the production function $F_t(L, K) = F(A_t L, K)$, where A_t grows at the exogenous rate $\widehat{A}$. Again, assume that we start at (or close to) a steady state of the laissez-faire system. At such a state $k_t = \frac{K_t}{A_t \overline{L}} = k^*$is a constant determined by $\frac{sf(k^*)}{k^*} - \delta = \widehat{A}$ (i.e. K_t grows like A_t).

As before an minimum wage is introduced at $t = 0$. We first consider the benchmark case that, once introduced, the minimum wage growth at the rate of labor productivity growth $\widehat{A}$. We denote by $\widetilde{w}_t$ the minimum wage *per labor efficiency unit*. Thus, in the benchmark case $\widetilde{w}_0 > (w_0/A_0)$ and $\widetilde{w}_t = \widetilde{w}_0$. As before, marginal productivity of labor matches minimum wages

$$F_L(\widetilde{L}_t, \widetilde{K}_t) = A_t \left(f(\widetilde{k}_t) - f'(\widetilde{k}_t)\widetilde{k}_t \right) = A_t \widetilde{w}_t, \tag{2}$$

where $f(k) = F(1, k)$ and $\widetilde{k}_t = \frac{\widetilde{K}_t}{A_t \widetilde{L}_t}$. Thus, $\widetilde{k}_t = \widetilde{k}_0$ is constant as before. Since $(\widetilde{w}_0/A_t) > w_0$, we have $\widetilde{L}_0 < \overline{L}$, $\widetilde{k}_t > k_0$ and therefore $\widehat{\widetilde{K}}_t = \widehat{\widetilde{K}}_0 = \frac{sf(\widetilde{k}_0)}{\widetilde{k}_0} - \delta < \widehat{K}^* = \widehat{A}$. Thus, since $\widetilde{k}_t = \frac{\widetilde{K}_t}{A_t \widetilde{L}_t}$, $\widetilde{L}_t$ has to fall at the constant rate $\widehat{A} - \widehat{\widetilde{K}}_0$. Employment falls at the rate equal to the difference of the (exogenous and constant) rate of technical progress and the (constant) rate of accumulation.

Proposition 5. *Starting from a steady state in the laissez-faire with exogenous technological progress, the introduction of a constant minimum wage in labor efficiency units initially reduces employment, reduces the rate of accumulation to a level below the laissez-faire rate of accumulation, and induces a continues fall of employment at a rate equal to the difference of the growth rate of labor productivity and the rate of accumulation.*

As before the conclusion is extreme. In the benchmark case employment will continue to fall after the initial decline. This conclusion is only strengthened if the minimal wage rises faster than in labor productivity. As before one may therefore expect that it may be impossible to implement rising minimum wages over extended periods of time even if labor-productivity grows at the same rate.

Let $w^*(K_t/A_t)$ be the market clearing wage (i.e. $L_t = \overline{L}$) at a given capital stock per labor efficiency unit.

Corollary 6. *Assume that there exists a* $\Delta > 0$ *such that in all periods* $\frac{w^*(K/A)}{w} > \Delta$, *where* w *is the actual wage and* K/A *the actual capital stock per labor efficiency unit. Then, an initial minimum wage, which after introduction rises at least at the rate of labor productivity growth, is not sustainable in the long-run.*

Proof: Corollary of proposition 5, since $\frac{w^*(K_t/A_t)}{A_t \widetilde{w}_t}$ tends to zero, for any minimum wage per efficiency unit $\widetilde{w}_t$, exceeding the steady state per efficiency unit market clearing wage.

So far we only dealt with minimal wages that rise at least as fast as labor productivity does. Given that such minimum wage regimes are typically unsustainable in the long run one may wonder what happens if minimum wages rise at a lower rate.

Proposition 7. *Starting from a steady state in the laissez-faire with exogenous technological progress, a minimum wage, which after introduction is raised at a rate lower than labor productivity growth, initially reduces employment and increases the rate of accumulation. After a while employment starts rising again until full employment is reestablished. At this time the minimum wage ceases to be binding, the actual wage per efficiency unit and the capital stock per efficiency unit are* lower *than their initial laissez-faire steady state level.*

Proof: The initial wage per efficiency unit, $\left(f(\widetilde{k}_0) - f'(\widetilde{k}_0)\widetilde{k}_0\right)$ is larger than the steady state wage in efficiency units, $(f(k^*) - f'(k^*)k^*)$. Therefore $\widetilde{k}_0$ is larger than k^*. The minimum wage growth at a lower rate than labor productivity, i.e. $\widetilde{w}_t$ falls. Therefore $\widetilde{k}_t$ falls. Consider the rate of change in employment, $\widehat{\widetilde{L}}_t = \widehat{\widetilde{K}}_t - \widehat{A} - \widehat{\widetilde{k}}_t$. As long as $\widetilde{k}_t > k^*$, $\widehat{\widetilde{K}}_t - \widehat{A}$ is negative, at $\widetilde{k}_t = k^*$, $\widehat{\widetilde{K}}_t - \widehat{A}$ is zero and it is positive for $\widetilde{k}_t < k^*$. Since $\widehat{\widetilde{k}}_t$ is negative, $\widehat{\widetilde{L}}_t$ becomes strictly positive before $\widetilde{k}_t$ has reached k^* and thereafter remains strictly positive as long as full employment is not reestablished. Thus full employment will be reestablished, say at time T.

We now show, that $\widetilde{k}_T < k^*$. Suppose not, i.e. suppose $\widetilde{k}_T \geq k^*$. Then, $\widetilde{k}_T = \frac{\widetilde{K}_T}{A_T \overline{L}} > (\frac{K}{A})^* \frac{1}{\overline{L}} = k^*$. But $\frac{\widetilde{K}_t}{A_t}$ has continuously fallen since $\widetilde{k}_t \geq k^*$, for all $t \in [0, T]$. Therefore, $\frac{\widetilde{K}_T}{A_T} < (\frac{K}{A})^*$, which is a contradiction.

From $\widetilde{k}_T < k^*$ it follows that the market clearing wage per efficiency unit given $\frac{\widetilde{K}_T}{A_T}$ is smaller than the steady state wage in efficiency units, that is, the initial wage before the minimum wage was introduced. After time T the system behaves as in the laissez-faire regime. The minimum wage at time T equals the market clearing wage $w^*(K_T/A_T)$, thus ceases to be binding and never becomes binding again (unless an adjustment occurs). ■

3. Unemployment in a model with induced change and without capital accumulation

In the previous section we have looked at the cases of pure accumulation and of exogenous technological change. We now consider the opposite extreme case without capital accumulation but endogenous direction of change.

3.1. The model of induced innovation. We first sketch the model of induced innovation in a laissez-faire world with fully flexible wages, which has been analyzed in the literature ([5], [8], [11], [12]).

The linear homogenous aggregate production function, $F_t(L,K)$, *given* the state of technological knowledge does not much differ from the one of the previous section. The dependency of the production function on time takes the specific form of 'factor-augmenting technical progress', formally $F_t(L,K)$ can be written as $F_t(L,K) = F(A_tL, B_tK)$, where A_t and B_t are technological parameters. In standard neoclassical growth theory, these parameters are exogenous. Furthermore, B_t is assumed to be constant in standard growth theory, that is, technical progress is *assumed* to be purely labor-augmenting. In the present model of induced innovation, both A_t *and* B_t can change with time and their evolution is determined endogenously.

The central hypothesis is the Hypothesis of induced innovation (see for instance [5] or [11], for a microeconomic foundation see [6]). It assumes that firms choose the instantaneous rates of factor augmentation $(\widehat{A}_t, \widehat{B}_t)$ such as to maximize the instantaneous growth rate of aggregate output $F(A_tL, B_tK)$ at each moment of time subject to an innovation possibility frontier $\left\{(\widehat{A}_t, \widehat{B}_t) \in \Re^2 | \widehat{B}_t \leq \eta(\widehat{A}_t)\right\}$, taking as given (L,K), (where $\frac{L}{K} = \frac{\overline{L}_t}{K_t}$). The function $\eta(\cdot)$ defining the possibility frontier is time independent, decreasing, and strictly convex. Taking into account that the rate of aggregate output growth can be written as (see [5])

$$\widehat{Y}_t = \frac{w_t\overline{L}}{Y_t}\widehat{A}_t + \frac{r_tK_t}{Y_t}\widehat{B}_t,$$

maximization of $\widehat{Y}_t$ given the innovation possibility frontier directly yields a sufficient condition uniquely determining $\widehat{A}_t$ (and $\widehat{B}_t = \eta(\widehat{A}_t)$):

$$(-\eta') = \frac{r_tK_t}{w_t\overline{L}} \tag{3}$$

Wages w_t and capital rental rates r_t (both in terms of output) are equal to the marginal productivities of the factors at L_t and K_t. We assume that the production function is a simple CES function, $F(A_tL, B_tK) = [(A_tL)^\rho + (B_tK)^\rho]^{(1/\rho)}, \rho < 1$. The relevant feature of this specification is that the algebraic sign of $(1-\sigma)$ is constant on the full domain of the production function, where σ is the elasticity of substitution. In the CES case, marginal productivity pricing at market clearing quantities yields the relation $\frac{w_t}{r_t} = \left(\frac{A_t}{B_t}\right)^{\rho} \left(\frac{\overline{L}}{K_t}\right)^{\rho-1}$ and hence $\frac{r_t K_t}{w_t \overline{L}} = \left(\frac{B_t K_t}{A_t \overline{L}}\right)^{\rho}$, which allows to rewrite condition (3) as

$$(-\eta') = \left(\frac{B_t K_t}{A_t \overline{L}}\right)^{\frac{1-\sigma}{\sigma}} = (k_t)^{\frac{1-\sigma}{\sigma}}, \tag{4}$$

where $k_t = \frac{B_t K_t}{A_t \overline{L}}$ and where $\sigma = \frac{1}{1-\rho}$ is the elasticity of substitution. This relations uniquely determine the rates of factor augmentations $(\widehat{A}_t, \widehat{B}_t)$ as a function of the current ratio of factor shares or of the ratio of factor supplies in terms of efficiency units.

The dynamics under laissez-faire. Condition (4) together with the rule of capital accumulation govern the dynamics of the system. In the present section we consider the case of two factors that do not accumulate. L and K may be thought of as labor and land or as unskilled labor and skilled labor, for instance. This is the framework of [11]. The dynamics is completely determined by condition (4), then. Recalling the properties of the transformation curve $\eta(\cdot)$, one easily sees that there is a unique k^* at which $\frac{B_t}{A_t}$ is constant $(\widehat{A}_t = \widehat{B}_t)$. For the case that the two factors are good complements $(\sigma < 1)$ $\frac{B_t}{A_t}$ falls $(\widehat{A}_t > \widehat{B}_t)$ when $k_t > k^*$ and rises when $k_t < k^*$. In the laissez-faire case without accumulation (K and L constant) this means that the steady state of the reduced system with the ratio k^* is globally stable if $\sigma < 1$. Analogously, if the factors are good substitutes, i.e. if $\sigma > 1$, the steady state is unstable. We follow the literature in restricting attention to the stable case.

3.2. The dynamics in the minimum wage regime. Again starting at (or close to) a steady state of the laissez-faire economy, we want to know the effect of an introduction of a minimum wage exceeding the market clearing wage. As in the previous section we first consider the benchmark case in which the minimum wage, once introduced, is raised continuously at the rate of labor productivity growth, i.e. the case of a constant minimum wage per labor efficiency unit.

Condition (3) was derived given current prices and current state of knowledge. For innovators it is not relevant whether the factor-prices they actually have to pay, clear markets or not. The condition remains relevant in the minimum wage regime. Furthermore, factor-prices equal marginal productivities also in the minimum wage regime. Therefore, condition (4) remains valid as well. Thus we have

Lemma 8. *The rates of factor-augmentation $(\widehat{\widehat{A}}_t, \widehat{\widehat{B}}_t)$ are fully determined by k_t. In particular, $(\widehat{\widehat{A}}_t, \widehat{\widehat{B}}_t)$ is constant if k_t is constant.*

As in the previous cases fixing the wage in efficiency units above its competitive level indirectly fixes the ratio of employed quantities in efficiency units. Equation (2) remains valid, for $\widetilde{k}_t = \frac{B_t \widetilde{K}_t}{A_t \widetilde{L}_t}$. Thus, $\widetilde{k}_t = \widetilde{k}_0$ is constant as before, as long as the minimum wage is sustained.

Lemma 9. *As long as the benchmark minimum wage is binding, the rates of factor augmentations, $(\widehat{\widehat{A}}_t, \widehat{\widehat{B}}_t)$, are constant.*

Since $\widetilde{w}_0 > w_0/A_t$, we have $\widetilde{L}_0 < \overline{L}$, $\widetilde{k}_0 > k_0$. At k_0 we had $\widehat{A}_0 = \widehat{B}_0$. Therefore, since $\widetilde{k}_t = \widetilde{k}_0 > k_0$, we have $\widehat{\widehat{A}} = \widehat{A}_t > \widehat{B}_t = \widehat{\widehat{B}}$ as long as the minimum wage is sustained. There is a continuous bias towards labor-augmentation. The minimum wage induces a constant *(Hicks-)automation bias* $\widehat{\widehat{A}} - \widehat{\widehat{B}} > 0$. Thus, since $\widetilde{k}_t = \frac{B_t K_t}{A_t \widetilde{L}_t}$, $\widetilde{L}_t$ has to fall at the constant rate $\widehat{\widehat{A}} - \widehat{\widehat{B}}$. Employment falls at the rate of the automation bias.

Proposition 10. *Starting from the steady state in the laissez-faire economy without accumulation, the introduction of a constant minimum wage in labor efficiency units reduces initial employment, induces a bias towards automation at a constant rate and a continues fall of employment at the rate of automation.*

If the minimum wage is increased faster than in the benchmark case, than employment will fall faster than in the benchmark case. As in the previous section such minimum wages are typically unsustainable. On the other hand if minimum wages are increased at a lower speed than labor productivity the economy will find back after a while to full employment:

Proposition 11. *Starting from a steady state in the laissez-faire regime, a minimum wage, which after introduction is raised at a rate lower than labor productivity growth, initially reduces employment and increases the rate of automation. After a while employment starts rising again until full employment is reestablished. At this time the minimum wage ceases to be binding, the actual wage per efficiency unit and the degree of automation (A/B) are* lower *than their initial laissez-faire steady state level.*

Proof: The proof is analogous to the one of Proposition 7. The initial wage per efficiency unit, $\widetilde{w}_0 = \left(f(\widetilde{k}_0) - f'(\widetilde{k}_0)\widetilde{k}_0\right)$ is larger than the steady state wage in efficiency units, $(f(k^*) - f'(k^*)k^*)$. Therefore $\widetilde{k}_0$ is again larger than k^*. Furthermore $\widetilde{w}_t$ falls, therefore $\widetilde{k}_t$ falls. The rate of change of employment now is, $\widehat{\widetilde{L}}_t = (\widehat{\widehat{B}}_t - \widehat{\widehat{A}}_t) - \widehat{\widetilde{k}}_t$. As long as $\widetilde{k}_t > k^*$, $\widehat{\widehat{B}}_t - \widehat{\widehat{A}}_t$ is negative, at $\widetilde{k}_t = k^*$, $\widehat{\widehat{B}}_t - \widehat{\widehat{A}}_t$ is zero and it is positive for $\widetilde{k}_t < k^*$. Since $\widehat{\widetilde{k}}_t$ is negative, $\widehat{\widetilde{L}}_t$ becomes strictly positive before $\widetilde{k}_t$ has reached k^* and thereafter remains strictly positive as long as full employment is not reestablished (since $\widetilde{w}_t$ and therefore $\widetilde{k}_t$ continue to fall). Thus full employment will be reestablished, say at time T.

We now show, that $\widetilde{k}_T < k^*$. Suppose not, i.e. suppose $\widetilde{k}_T \geq k^*$. Then, $\widetilde{k}_T = \frac{\widetilde{B}_T K}{\widetilde{A}_T \overline{L}} \geq \left(\frac{B}{A}\right)^* \frac{K}{\overline{L}} = k^*$. But, starting at $\frac{\widetilde{B}_0}{\widetilde{A}_0} = (\frac{B}{A})^*$, $\frac{\widetilde{B}_t}{\widetilde{A}_t}$ has continuously fallen since $\widetilde{k}_t > k^*$, for all $t \in [0, T)$. Therefore, $\frac{\widetilde{B}_T}{\widetilde{A}_T} < (\frac{B}{A})^*$, which is a contradiction.

From $\widetilde{k}_T < k^*$ it follows that the market clearing wage per efficiency unit given $\frac{\widetilde{B}_T}{\widetilde{A}_T}$ is smaller than the steady state wage in efficiency units, that is, the initial wage before the minimum wage was introduced.

After time T the system behaves as in the laissez-faire regime. The minimum wage at time T equals the market clearing wage $w^*(B_T/A_T)$, thus ceases to be binding and never becomes binding again. ∎

In view of the above propositions, one may wonder whether there is a minimal wage regime that leads to a new steady state with constant and strictly positive employment below full employment (k_t constant, $L_t = L$ constant, with $0 < L < \overline{L}$). As the following proposition shows there is in fact a very natural regime which leads to such a steady state. Suppose again that a minimal wage is introduced at the steady state of the laissez-faire economy. After introduction it is now raised at the rate that everybody in the economy was used to untill then, i.e. the laissez-faire steady state rate of labor productivity growth $\widehat{A}(k^*)$.

Proposition 12. *Starting from a steady state in the laissez-faire regime, a minimum wage, which after introduction is raised at the laissez-faire steady state rate of wage augmentation, initially reduces employment and increases the rate of automation. Thereafter, wages in efficiency units fall continuously to approach their initial level. The system approaches a new steady state with Hicks neutral progress, a lower than initial degree of automation (A/B), and a constant and strictly positive level of employment below full employment.*

Proof: 1) The initial wage increase increases $\widetilde{k}_0 = \frac{\widetilde{B}_0}{\widetilde{A}_0}\frac{K}{\widetilde{L}_0}$. Hence employment is reduced and $\widehat{A}(\widetilde{k}_0) > \widehat{A}^*$. Therefore, $\widehat{\widetilde{w}}_0 < 0$. The wage in efficiency units continuous to fall as long as $\widetilde{k}_t > k^*$.

2) Since $\widehat{\widetilde{w}}_0$ approach zero as $\widetilde{k}_t$ approaches k^*, $\widetilde{k}_t$ falls and $\widehat{\widetilde{k}}_t$ approaches zero as $\widetilde{k}_t$ approaches k^*.

3) At $t = 0$, $\frac{\widetilde{B}_0}{\widetilde{A}_0} K_0 = (\frac{B}{A} K)^*$. Thereafter $\frac{\widetilde{B}_t}{\widetilde{A}_t} K$ falls continuously at a rate approaching 0. Thus $\lim \frac{\widetilde{B}_t}{\widetilde{A}_t} K < (\frac{B}{A} K)^*$. But $\lim_{t\to\infty} \frac{\widetilde{B}_t}{\widetilde{A}_t}\frac{K}{\widetilde{L}_t} = \lim_{t\to\infty} \widetilde{k}_t = k^* = \left(\frac{B}{A}\right)^* \frac{K}{\overline{L}}$. Thus $\lim \widetilde{L}_t < \overline{L}$. ∎

Note, that the system can only be in a steady state with k_t and L_t constant, if $k_t = \frac{B_t}{A_t}\frac{K}{L_t} = k^*$, since is $\frac{B_t}{A_t}K$ constant only at $k_t = k^*$. There is such a steady state for any level of employment $0 < L < \overline{L}$, choosing $\frac{B_t}{A_t}K = k^* L$. Minimal wages in efficiency units are at the laissez-faire steady state level. Which of these steady states the system approaches in the above proposition depends on the extent of the initial wage increase.

4. UNEMPLOYMENT IN A MODEL WITH INDUCED CHANGE AND CAPITAL ACCUMULATION

We now turn to the complete model with both accumulation and induced innovation.

4.1. The dynamics under laissez-faire. We first describe the dynamics of the system in the laissez-faire economy. In doing so we stick to the version of Drandakis and Phelps [5] so that the aggregate variables in our model evolve exactly as in theirs. The instantaneous rate of accumulation simply is a constant fraction s of current aggregate output $Y_t = F(A_t\overline{L}, B_t K_t)$ and there is no depreciation, i.e. $\dot{K}_t = sF(A_t\overline{L}, B_t K_t)$. Then

$$\widehat{k}_t = (\widehat{B}_t - \widehat{A}_t) + s\frac{F(1, k_t)}{k_t}B_t. \tag{5}$$

As we have seen, the variables $\widehat{A}_t$ and $\widehat{B}_t$ are fully determined by k_t. Thus, k_t (and therefore the factor shares) can only be constant if B_t is constant. As in the neoclassical model without endogenous innovation factor shares can only remain constant if progress is purely labor augmenting. Due to equation (5) we can completely describe the dynamics by the evolution of the two variables k_t and B_t:

$$\begin{cases} \dot{k}_t = & (\widehat{B}(k_t) - \widehat{A}(k_t))k_t - sF(1, k_t)B_t \\ \dot{B}_t = & \widehat{B}_t(k_t)B_t, \end{cases} \tag{6}$$

where $\widehat{A}(k_t)$ is the solution to equation (4) and $\widehat{B}(k_t) = \eta(\widehat{B}(k_t))$.

Since the system is the same as that of [5] we only state the results about its behavior without deriving them. Under the specifications we made so far the system has a unique steady state (i.e. a (k_t, B_t) for which $(\dot{k}_t, \dot{B}_t) = (0, 0)$). Since in such a steady state progress is purely labor augmenting, the steady has all the properties of the steady state of in the neoclassical model with exogenous technological change. The steady state is globally stable if the factors are good complements ($\sigma < 1$) and the steady state is globally unstable if the factors are good substitutes ($\sigma > 1$). In the former case can the assumption of purely labor augmenting change be explained on the basis of the induced innovation hypothesis. In what follows we only consider the case in which the steady state of the laissez-faire system is stable.

4.2. The dynamics in the minimum wage regime. Then introducing a minimum wage we first stick to the bench-mark case in which the minimum wage, once introduced, rises at the rate of labor productivity growth. To analyze this case we only have to put together the insides from the two previous sections. At $t = 0$ a minimum wage $\widetilde{w}_0 > w_0$ is introduced and raised at the rate $\widehat{\widetilde{w}}_t = \widehat{A}_t$. As before the ratio of quantities in efficiency units $\widetilde{k}_t = \frac{B_t \widetilde{K}_t}{A_t \widetilde{L}_t}$ remains constant at its level $\widetilde{k}_0$ as long as the minimum wage remains binding. Independently from initial conditions $0 = \widehat{\widetilde{k}}_t = \widehat{\widetilde{K}}_t - \widehat{\widetilde{L}}_t - \left(\widehat{\widetilde{A}}_t - \widehat{\widetilde{B}}_t\right)$. Therefore, independently of the initial conditions we always have the following lemma:

Lemma 13. *As long as the minimum wage is binding employment changes at a rate equal to the difference between the rate of accumulation and the rate of automation,* $\widehat{\widetilde{L}}_t = \widehat{\widetilde{K}}_t - \left(\widehat{\widetilde{A}}_t - \widehat{\widetilde{B}}_t\right)$.

Since the rate of factor augmentation only depends on $\widetilde{k}_t$ Lemma 9 still holds. The rates of factor augmentations, $(\widehat{\widetilde{A}}_t, \widehat{\widetilde{B}}_t)$, are constant as long as the minimum wage is binding.

Whether the minimum wage remains binding in the long-run depends on whether the rate of change of employment after the initial reduction of employment is positive or negative. This in turn depends on the state of the laissez-faire economy at time $t = 0$, at which the minimum wage is introduced.

To describe the dynamics of the system with minimum wage depending on the state of the laissez-faire economy we look at each of the four regions of the phase diagram of the laissez-faire dynamic system for the variables k and B , defined by (6). This is done in the appendix.

The following proposition summarizes the long-run consequences of a minimum wage in the benchmark case :

Proposition 14. *If at the time of the introduction of the minimum wage, there is less capital (in efficiency units) per efficiency unit of labor than at the steady state of laissez-faire ($\widetilde{k}_0 < k^*$), then the system always moves back to the laissez-faire. Minimal wages are not binding in the long-run.*

If $\widetilde{k}_0 > k^*$ and $B_0 < B(\widetilde{k}_0)$, in particular, if the system starts close to the steady state of the laissez-faire regime, then the minimum wage remains binding for ever. The labor-share of aggregate output is raised. Technological change is Harrod labor-saving ($(\widehat{A} - \widehat{B}) - \widehat{K}) > 0$), with a constant rate of automation $(\widehat{A} - \widehat{B})$. The rate of accumulation tends to zero. Employment falls at a rate equal to the difference between the rate of automation and the rate of accumulation. Per capita output and consumption tend to zero.

Note that the effect of decreasing capital accumulation on the reduction of employment is dampened by the automation bias. In the case without endogenous technological change, the stock of capital falls at a constant rate. In the case with endogenous technological change, this rate falls at the same rate as B_t.

We have already noted that the strong consequences of a minimum wage (for the benchmark case of constant minimum wage in efficiency units) make it unlikely that minimum wages in efficiency units can be hold above marginal productivity. As before we can formalize this in the following assumption:

Assume that there exists a $\Delta > 0$ such that $\frac{w^*(\frac{BK}{A})}{w} > \Delta$, where $w^*(\frac{BK}{A})$ is the market clearing wage, given $\frac{BK}{A}$ and where w is the actual wage. Then the following Corollary is a direct consequence of proposition 14.

Corollary 15. *A fixed minimum wage (in efficiency units) is either not binding in the long run or is not sustainable in the long-run. In particular, if it is introduced at the steady state of the laissez-faire regime it is not sustainable.*

As we have seen the effect of a slow rising minimum wage (i.e. one that, once introduced, is increased at a *lower* rate than the rate of labor productivity growth) on the evolution of employment, was similar in the two polar cases of the previous sections (although the mechanism causing the effect is very different in the two cases). Not surprisingly the effect of a slow rising minimum wage in the complete model leads to the same conclusion.

Proposition 16. *Starting from a steady state in the laissez-faire regime, a minimum wage, which after introduction is raised at a rate lower than labor productivity growth, initially reduces employment, increases the rate of automation, and decreases the rate of accumulation. After a while employment starts rising again until full employment is reestablished. At this time the minimum wage ceases to be binding, the actual wage per efficiency unit and the ratio of capital to labor in efficiency units are* lower *than their initial laissez-faire steady state levels.*

Proof: Analogous to the proofs of Propositions 7 and 11. The term $\widehat{\widetilde{B}}_t - \widehat{\widetilde{A}}_t$ of Proposition 11 should be replaced by the term $\widehat{\widetilde{K}}_t - (\widehat{\widetilde{A}}_t - \widehat{\widetilde{B}}_t)$, $\frac{\widetilde{B}_t}{\widetilde{A}_t}$ by $\frac{\widetilde{B}_t \widetilde{K}_t}{\widetilde{A}_t}$. ■

Similarly, if minimal wages, after introduction, are raised at the experienced laissez-faire steady state rate of growth, the system approaches a new steady state with constant positive unemployment and binding minimal wages.

Proposition 17. *Starting at a steady state in the laissez-faire regime, a minimum wage, which after introduction is raised at the old steady state rate of wage growth, initially reduces employment and induces Harrod labor-saving progress. Thereafter,* *wages in efficiency units fall continuously and approach their initial steady state level. The system approaches a new steady state with Harrod neutral progress and a constant and strictly positive level of unemployment.*

Proof: As proof of Proposition 12

5. CONCLUSIONS

We can now answer the questions posed in the introduction. The introduction of the minimal wage reduces employment and increases firms' effort to save labor. The resulting automation bias and decrease in capital accumulation initially further reduces employment. The long-run consequences of a minimum wage depend on how the minimum wage, once introduced, is changed over time. The case of constant minimum wages in labor efficiency units (i.e. minimum wages that, once introduced, are raised at the rate of the growth of labor productivity) provides a natural benchmark: Fixing wages in in efficiency units fixes the ratio of capital to labor in efficinecy units, $k = \frac{BK}{AL}$, which much simplifies the analysis.

If, starting at the steady state of the laissez-faire regime, an initial minimum wage per labor efficiency unit remains constant, then there results a constant Hicks-automation-bias (the difference of the rate of automation $\widehat{A}_t - \widehat{B}_t$, under minimum wages and under laissez-faire) and a decreasing rate of accumulation. In the case without accumulation, progress becomes Hicks-labor-saving (without a minimal wage progress is Hicks-neutral). In the case with accumulation (and endogenous technological change) progress becomes Harrod-labor-saving (without a minimal wage progress is Harrod-neutral). Employment, after an initial decrease, falls at a rate equal to the difference of the Hicks-automation-bias and the rate of accumulation. Labor's share is persistently (level effect), although labor incomes fall continuously.

These conclusions are accentuated if the minimal wage per efficiency unit, once introduced, is further raised. If, on the other hand, the initial minimum wage increases at a rate smaller than labor-productivity growth, then the economy will find its way back to the laissez-faire regime. Before it does so the minimum wage in labor efficiency units, which initially was raised above the laissez faire steady state level, will have fallen below this level. An initial increase of the rate of automation, decrease of the rate of accumulation, and increase in labor's share will be reversed after a while, before the laissez-faire regime with full employment will again be reached at lower than steady state wages, lower ratio of capital stock in efficiency units to labor in efficiency units, and at a lower degree of automation.

The benchmark minimum-wage regime (constant minimum wage per efficiency unit) is unsustainable in an economy in which the ratio of market-clearing wage to actual wages is bounded away from zero. If the rising unemployment leads to a reduction of the minimum wage in efficiency units back to the initial laissez-faire steady state wage, then the economy reaches a new steady state with a constant strictly positive level of unemployment. Similarly, if the minimum wage is increased at the rate at which wages used to increase under laissez-faire, the system will automatically approach a new steady state with a constant level of unemployment.

In the present paper we have analyzed economies with only two factors, of which one may or may not accumulate. In the case of two non-accumulating factors, L may be unskilled labor and K skilled labor. Formally, the minimum wage of the present paper has only be imposed on unskilled labor. This is equivalent to imposing a minimum wage on both types of labor if we consider cases in which the minimum wage is never binding for skilled labor. Nevertheless, the issue of skill, unemployment and biased innovation cannot be addressed in full in a model with only two factors. In future work we plan to analyze the dynamics of innovation, wage, and employment in the case of three factors: unskilled labor, skilled labor, and capital. Skilled labor is a complement to capital (which accumulates) and unskilled labor is a substitute to the two other factors.

Needless to say, our results depend to a certain degree on the specifics of the analyzed model of accumulation and induced innovation. Nordhaus [9] and Binswanger [3] criticize the postulation of an exogenous and stationary innovation-frontier. The objections concern the specific assumptions of the model and their consequences rather than the approach of induced innovation (see [6]). While the characterization of the laissez-faire steady state (Harrod-neutral in the model with accumulation) depends on the stationarity of the innovation frontier (see [9]), one can expect that the direction of the deviation of the dynamics with minimum wages from the laissez-faire dynamics is robust with respect to departures from the benchmark case of a stationary innovation frontier.

6. Appendix

6.1. Alternative saving rules. We have assumed in the main text that at each point of time a constant fraction of aggregate output is saved and turned into new capital. The decisive aspect of this saving rule is that savings are an increasing function of aggregate output .We show in the model of pure accumulation, that the conclusions remain valid for alternative assumptions on the propensity to save.

Endogenous propensity to save

Consider the framework of section 2. Independently of the saving rule, the ratio capital to labor, $\widetilde{k}_t$, and therefore the interest rate, are constant as long as minimum wages are binding (Lemma 1). A consumer who foresees a continuous fall in income and a constant interest rate, so one may think, would want to increase his savings to prevent income from tending to zero. This would increase capital stock and invalidate the above argumentation. This intuition ignores the fact that the constant interest rate after the wage increase is lower than at the laissez-faire steady state. We show that in the standard textbook framework of the model with endogenous saving, this leads to the same conclusion as in the case with fixed propensity to save.

We assume the standard representative consumer model (as for instance used in [2]). Unemployment means that the representative consumer works less than $\overline{L}$ (Alternatively one may assume that a fraction of consumers are fully unemployed, while the others work full time and that unemployment insurance equalizes the real income of the employed and the unemployed). The consumer chooses his consumption path $C(t)$ to maximizes his intertemporal utility

$$\int_0^\infty \frac{[C_t]^{(1-\theta)} - 1}{(1-\theta)} e^{-\rho t} dt$$

subject to the budget constraint $\int C_t e^{-\overline{r}(t)t} dt = K(0) + \int w_t L(t) e^{-\overline{r}(t)t} dt$, where $\overline{r}(t)$ is the average interest rate between time 0 and time t, $\overline{r}(t) = (1/t)\int_0^t r_\tau d\tau$. A necessary condition for solving the consumers' problem is that the rate of change of consumption $\widehat{C}_t = \frac{r_t - \rho}{\theta}$ at any moment of time. Assuming that the production function satisfies the Inada conditions the economy under laissez-faire posses a globally stable steady state with constant output, capital stock and consumption. Thus at this state $0 = \widehat{C}_t = \frac{r_t - \rho}{\theta} = \frac{(f'(k^*)-\delta)-\rho}{\theta}$. When the minimum wage is introduced in a laissez-faire economy at its steady state, the new capital-labor ratio, $\widetilde{k}$, is constant and $\widetilde{k} > k^*$as long as the minimum wage is binding. Therefore $\widehat{\widetilde{C}}_t = \frac{\widetilde{r}_t - \rho}{\theta} = \frac{(f'(\widetilde{k})-\delta)-\rho}{\theta} < \frac{(f'(k^*)-\delta)-\rho}{\theta} = \widehat{C}_t = 0$. Optimal consumption falls as long as the minimum wage is binding. We want to conclude from here that capital and employment fall and tend to zero, and that minimum wages do in fact remain binding.

Suppose first that the stock of capital rises at a rate strictly larger than zero. Then employment rises at the same rate, so as to keep $\widetilde{k}$ constant. Thus after a while employment will reach full employment $\overline{L}$ again and the economy continuous to develop as under laissez-faire. It is well known that on the equilibrium consumption path of the laissez-faire economy $C_t > C^*$ if $k_t > k^*$. Therefore, at the time full employment is reestablished $C_t > C^*$, since $k_t = \widetilde{k} > k^*$. Since before this time consumption is falling, it follows that, after the introduction of the minimum wage until the laissez-faire steady state is reached again, consumption is always higher than C^*. This is impossible since initial capital stock is the same than at this steady state. Thus capital stock and employment can not rise at a rate strictly larger than zero until full employment is reached.

It is not difficult to see that capital and employment cannot approach constant growth rates. Suppose they did. Then, in the long-run, capital stock, employment, and output are constant. On the other hand consumption falls at a constant rate. Thus $\lim \widehat{K}_t = \lim \frac{F(L_t,K_t)-C^t}{K_t} = \lim \frac{F(L_t,K_t)}{K_t} = \lim \frac{f(k_t)}{k_t} = \frac{f(\widetilde{k})}{\widetilde{k}} > 0$. This contradicts$\widehat{K}_t$ tending to zero.

It follows that the stock of capital tends to zero. Minimal wages remain binding. Employment and output tend to zero.

Kaldorian propensities to save. The conclusions hold in the equally stylized environment in which workers consume all labor income, capitalists save a fixed proportion of capital income (or, more generally, in which savings are an increasing function of current capital incomes). The minimum wage initially reduces employment at given capital stock. This reduces the marginal product of capital and hence rental rates and (since the capital stock is given) capital incomes. Thus savings are reduced proportional to the reduction in rental rates. The capital stock declines. Thereafter the rental rate is fixed (since $\widetilde{k}_t$ is fixed at $\widetilde{k}_t$), thus capital incomes and savings are further reduced. Hence, as before, employment continues to fall.

Only workers save. A third traditional class of models are the Diamond type models in which workers (the young save, while capitalists (the old) do not save. In these models the validity of Proposition 2 depends on the elasticity of substitution between the two factors. Assume that workers (the 'young') save (with savings being an increasing function of their income), while capitalist (the 'old') consume. The minimum wage initially reduces employment. Whether or not this reduces wage incomes depends on the elasticity of substitution of the production function $F(\cdot)$. If labor and capital are good substitutes, then the decline in employment offsets the increase in wages. Wage incomes are reduced and savings and hence the stock of capital fall. To support the minimum wage, employment has to further fall. In this case the proposition holds again.

If, on the other hand labor and capital are bad substitutes, then total wage income initially increases, despite the fall of employment. Savings and capital stock rise, so that employment too rises again and tends back to full employment. Then the dynamics continue as under laissez-faire, starting with a capital stock higher than the steady state level.

6.2. Proof of Proposition 14. The four regions in the (k, B)-diagram are defined by the two lines for which either $\widehat{k} = 0$ or $\widehat{B} = 0$ under laissez-faire. The intersection of these two lines defines the steady state of the laissez-faire. The $(\widehat{k} = 0)-$line simply is a vertical line. The $(\widehat{B} = 0)-$line is the graph of the function $B(k_0) = \left(\widehat{B}(k_0) - \widehat{A}(k_0)\right) \frac{k_0}{sf(k_0)}$. Note that the function $B(\cdot)$ is increasing, with $B(k) = 0$ for some $k \in [0, k^*]$.

Region I. $(\widetilde{k}_0, \widetilde{B}_0)$ lies below the $(\widehat{k} = 0)-$line and at the right of the $(\widehat{B} = 0)-$line, i.e. $\widetilde{k}_0 > k^*, \widetilde{B}_0 < B(\widetilde{k}_0)$.

Region I is of particular interest as any initial state (k_0, B_0) close to the steady state of the laissez-faire economy leads to a $(\widetilde{k}_0, \widetilde{B}_0)$ in this region ($\widetilde{k}_0 > k^*$ and $B(\cdot)$

is strictly increasing).

Under the dynamics of the laissez-faire system $\widehat{k}_t = \widehat{K}_t - \left(\widehat{A}_t - \widehat{B}_t\right) < 0$ in this region. In the mixed system however, as long as the minimum wage is binding, $\widehat{\widetilde{k}}_t = 0$ and therefore $\widehat{\widetilde{L}}_t = \widehat{\widetilde{K}}_t - \left(\widehat{\widetilde{A}}_t - \widehat{\widetilde{B}}_t\right) < 0$, since the rates of change of K, A, and B depend on k only. As long as employment falls the minimum wage remains binding. Therefore, $\widetilde{k}_t$ remains constant at $\widetilde{k}_0$. Hence also $\widehat{\widetilde{B}}_t$ remains constant at $\widehat{B}(\widetilde{k}_0) < \widehat{B}(k^*) = 0$. Therefore, $(\widetilde{k}_t, \widetilde{B}_t)$ forever remains in Region I. Employment falls continuously at the rate $\widehat{\widetilde{K}}_t - \left(\widehat{A}(\widetilde{k}_0) - \widehat{B}(\widetilde{k}_0)\right)$. The rate of automation $\left(\widehat{A}(\widetilde{k}_0) - \widehat{B}(\widetilde{k}_0)\right)$ is constant and larger than in the laissez-faire (Harrod-neutral under laissez-faire, labor-saving even in the sense of Harrod now) and the rate of capital accumulation $\widehat{\widetilde{K}}_t = s\frac{f(\widetilde{k}_0)}{\widetilde{k}_0}\widetilde{B}_t$ falls with $\widetilde{B}_t$ at the constant rate $\widehat{B}(\widetilde{k}_0)$. Therefore, in the long-run employment falls at a rate close to the rate of automation. Output $A_t L_t f(\widetilde{k}_0)$ changes at the rate $\widehat{A}(\widetilde{k}_0) + \widehat{\widetilde{L}}^t = \widehat{B}(\widetilde{k}_0) + \widehat{\widetilde{K}}_t$ which tends to $\widehat{B}(\widetilde{k}_0) < 0$. Thus output tends to zero.

Region II. $(\widetilde{k}_0, \widetilde{B}_0)$ lies below the $(\widehat{k} = 0)-$line and at the left of the $(\widehat{B} = 0)-$line, i.e. $\widetilde{k}_0 < k^*, \widetilde{B}_0 < B(\widetilde{k}_0)$.

Under the dynamics of the laissez-faire system $\widehat{k}_t = \widehat{K}_t - \left(\widehat{A}_t - \widehat{B}_t\right)$ is negative in this region as in Region I. As before, since $\widehat{\widetilde{k}}_t = 0$, as long as the minimum wage is binding, we have $\widehat{\widetilde{L}}_t = \widehat{\widetilde{K}}_t - \left(\widehat{\widetilde{A}}_t - \widehat{\widetilde{B}}_t\right) < 0$. As long as employment falls the minimum wage remains binding. Therefore, $\widetilde{k}_t$ remains constant at $\widetilde{k}_0$. Hence also $\widehat{\widetilde{B}}_t$ remains constant, but now, since $\widetilde{k}_0 < k^*$, at $\widehat{B}(\widetilde{k}_0) > \widehat{B}(k^*) = 0$. Therefore, after a while $(\widetilde{k}_t, \widetilde{B}_t)$ will pass from Region II to Region III. Since then employment has fallen continuously. Per capita output $\frac{Y_t}{L} = f(\widetilde{k}_0)\widetilde{A}_t\widetilde{L}_t$ falls at the rate $\widehat{\widetilde{A}}_t + \widehat{\widetilde{L}}_t = \widehat{\widetilde{B}}_t + \widehat{\widetilde{K}}_t$.

Region III. $(\widetilde{k}_0, \widetilde{B}_0)$ lies above the $(\widehat{k} = 0)-$line and at the left of the $(\widehat{B} = 0)-$line, $\widetilde{k}_0 < k^*$, $\widetilde{B}_0 > B(\widetilde{k}_0)$. Under the dynamics of the laissez-faire system $\widehat{k}_t = \widehat{K}_t - \left(\widehat{A}_t - \widehat{B}_t\right) > 0$ in this region. In the minimum wage regime $\widehat{\widetilde{k}}_t = 0$ and therefore $\widehat{\widetilde{L}}_t = \widehat{\widetilde{K}}_t - \left(\widehat{\widetilde{A}}_t - \widehat{\widetilde{B}}_t\right) > 0$, employment rises again after the initial decline. As long as employment remains below full employment the minimum wage remains binding and $\widetilde{k}_t$ remains constant at $\widetilde{k}_0$. Hence $\widehat{\widetilde{B}}_t$ remains constant at $\widehat{B}(\widetilde{k}_0) < \widehat{B}(k^*) = 0$. Therefore, $(\widetilde{k}_t, \widetilde{B}_t)$ remains in Region III until employment reaches the level of full employment. Minimal wages are no longer binding then, and the economy continuous to develop under laissez-faire. The minimum wage will never again become binding (unless, of course it is readjusted) and the system continuous to behave as under laissez-faire.

Region IV. $(\widetilde{k}_0, \widetilde{B}_0)$ lies above the $\widehat{k} = 0$ line and at the right of the $\widehat{B} = 0$ line, i.e. $\widetilde{k}_0 > k^*$, $\widetilde{B}_0 > B(\widetilde{k}_0)$. Under the dynamics of the laissez-faire system $\widehat{k}_t = \widehat{K}_t - \left(\widehat{A}_t - \widehat{B}_t\right) > 0$ in this region. In the mixed system $\widehat{\widetilde{k}}_t = 0$ and therefore $\widehat{\widetilde{L}}_t = \widehat{\widetilde{K}}_t - \left(\widehat{\widetilde{A}}_t - \widehat{\widetilde{B}}_t\right) > 0$. As in Region II employment rises again after the initial decline. However, since $\widetilde{k}_0 > k^*$ we now have $\widehat{B}(\widetilde{k}_0) < \widehat{B}(k^*) = 0$. Thus, $\widetilde{B}_t$ declines as long as the minimum wage remains constant, hence $(\widetilde{k}_t, \widetilde{B}_t)$ moves south towards Region I. If employment reaches the full employment level $\overline{L}$ before the process has reached Region I, then minimum wages are no longer binding and the system continuous to move in the laissez-faire mode. In the long run it approaches the steady state (k^*, B^*). If, on the other hand, the process reaches Region I before rising employment has reestablished full employment, then the process continuous as in Region I. Employment falls again and approaches zero in the long run.

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